

Advection dispersion equations,
their solutions and applications for
water quality in river.

Important Constituents and their representative values

TABLE 14.2.1 Typical Influent Municipal Waste Concentrations

Constituent	Concentration, mg/L		
	Strong	Medium*	Weak
Solids, total	1200	720	350
Dissolved, total	850	500	250
Fixed	525	300	145
Volatile	325	200	105
Suspended, total	350	220	100
Fixed	75	55	20
Volatile	275	165	80
Settleable solids, mL/L	20	10	5
Biochemical oxygen demand, 5-day, 20° (BOD ₅ -20°)	400	220	110
Total organic carbon (TOC)	290	160	80
Chemical oxygen demand (COD)	1000	500	250
Nitrogen (total as N)	85	40	20
Organic	35	15	8
Free ammonia	50	25	12
Nitrites	0	0	0
Nitrates	0	0	0
Phosphorus (total as P)	15	8	4
Organic	5	3	1
Inorganic	10	5	3
Chlorides†	100	50	30
Alkalinity (as CaCO ₃)*	200	100	50
Grease	150	100	50

Effect of Treatment on Five Important Constituents

Effluent concentrations, mg/L, Total removal efficiencies*					
Scheme no.†	5-day BOD	Chemical oxygen demand	Total suspended solids	Total phosphorus	Total nitrogen
0 (Raw influent)	200(0%)	500(0%)	200(0%)	10(0%)	40(0%)
1	130(35%)	375(25%)	100(25%)	9(10%)	32(20%)
2	40(80%)	125(75%)	30(85%)	7.5(25%)	26(35%)
3	25(88%)	100(80%)	12(94%)	7(30%)	24(40%)
4	18(91%)	70(86%)	7(96%)	1(90%)	22(45%)
5	18(91%)	70(86%)	7(96%)	1(90%)	4(90%)
6	13(94%)	60(88%)	1(99.5%)	1(90%)	3(92%)
7	2(99%)	15(97%)	1(99.5%)	1(90%)	2(95%)

* Efficiencies for wastewater treatment are for the approximate BOD₅ concentration range of 100 ≤ BOD₅ ≤ 400 mg/L.

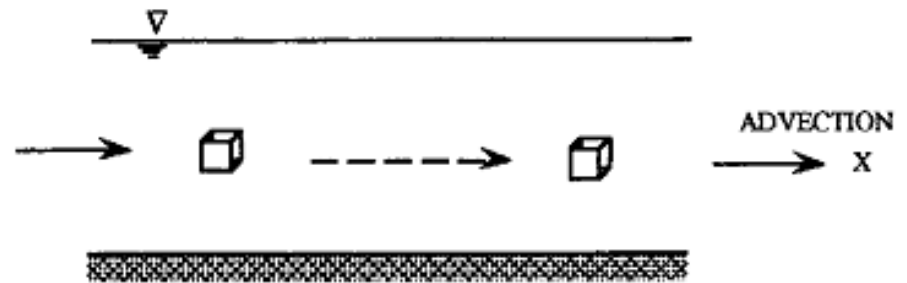
† Scheme no. Process

- 0 Wastewater influent (no treatment)
- 1 Primary treatment
- 2 1 + activated sludge (secondary treatment)
- 3 2 + polishing filter (high efficiency or super secondary)
- 4 3 + phosphorus removal and recarbonation
- 5 4 + nitrogen stripping
- 6 5 + pressure filtration
- 7 6 + activated carbon adsorption

Constituent Transport

- Flux, flow rate per unit area, or, quantity per unit area per unit time. ($\text{gm}/\text{m}^2\cdot\text{sec}$)
- Mechanism of Transport
 - ADVECTION (or CONVECTION)
 - DISPERSION

ADVECTION



- Advection (or Convection) is bodily transport due to the motion of fluid.
- The constituent moves with the fluid velocity
- Flux of constituents $F = UC$, U is velocity of flow, and C is concentration of constituents

For example, the mass flux of a constituent of concentration 2 g/m^3 in a fluid with velocity 3 m/s is $6 \text{ g/m}^2 \cdot \text{s}$. Similarly, the heat flux in water with temperature 25°C and velocity 3 m/s is

$$\begin{aligned} F &= U \rho c_p T \\ &= 3 \text{ m/s} \times 1000 \text{ kg/m}^3 \times 4.2 \times 10^3 \text{ J/kg} \cdot ^\circ\text{C} \times 25^\circ\text{C} \\ &= 3.15 \times 10^8 \text{ J/m}^2 \cdot \text{s} \end{aligned}$$

where ρ is the density and c_p is the heat capacity of water.

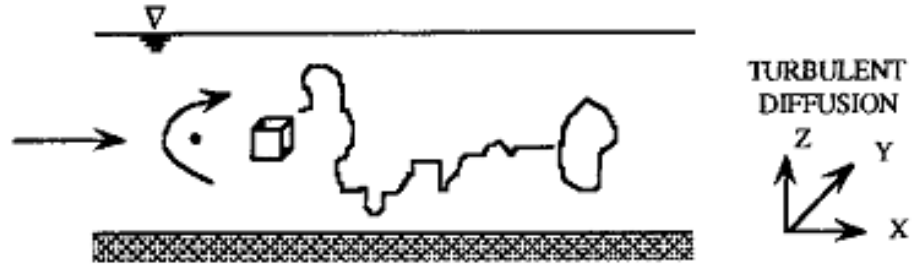
Advection does not alter the shape (distribution of concentration versus distance) of the constituent distribution as long as the velocity distribution remains uniform. The center of mass of the constituent distribution in water moves with the mean velocity.

Diffusion

- Diffusion is process of transport of constituents in the direction of decreasing concentration of the constituents
- Flux in case of diffusion is proportional to concentration gradient
- Fick's First Law of Diffusion: $F_x = -E_x \frac{\partial C}{\partial x}$

where concentration again should be expressed in units of quantity per volume and E_x is the diffusivity, with units of length²/time. Equation (14.3.2) is also known as Fick's first law of diffusion, and the negative sign indicates a positive flux in the direction of negative gradient (i.e., in the direction of decreasing concentration).

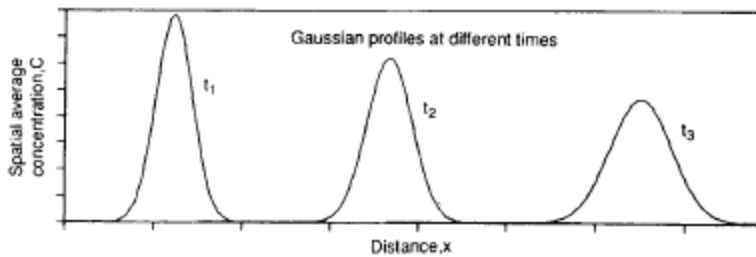
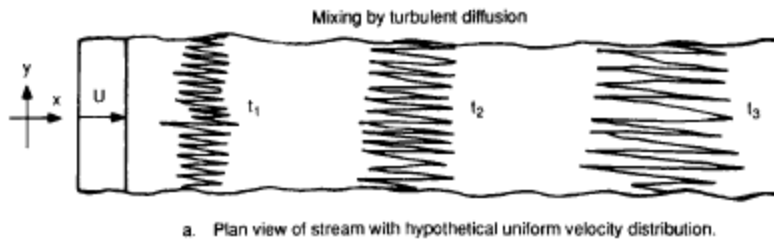
Diffusion



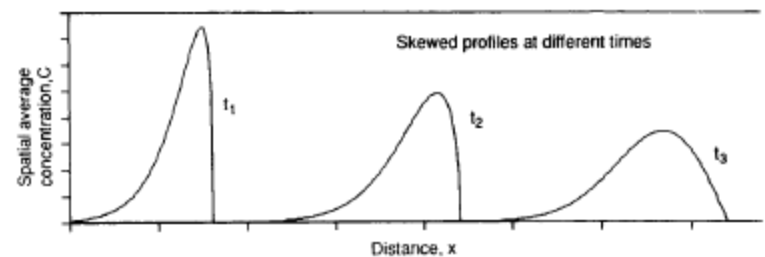
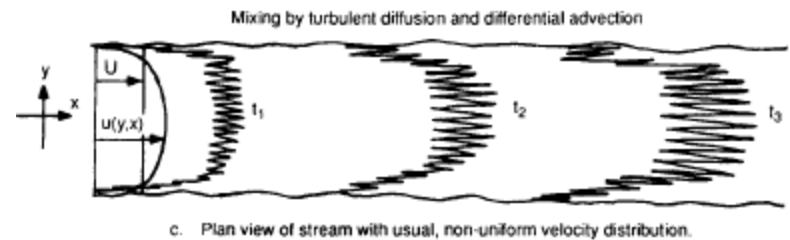
- Molecular Diffusion
 - It occurs in stagnant water or in laminar flow
 - It is of interest to chemist
- Turbulent Diffusion
 - It is of interest to river engineers, as turbulent flow is dominant
 - Turbulent Diffusivity in all direction is same for homogeneous turbulence, thus $E_x = E_y = E_z$
 - Turbulent Diffusivity is of order of 1 to 10,000 cm²/sec (0.001 to 10 ft²/sec)
 - Diffusivity is measured using tracer studies.
 - It is difficult to measure, so it is taken as Calibration Parameter.

Shear-Flow Dispersion

- It is diffusion type transport process
- It occurs due to non-uniform velocity distribution (variation of velocity)



Advection and Turbulent Diffusion Only
 U and C are average over the cross-section



Advection + Turbulent Diffusion + Shear Dispersion.
 U is non-Uniform, that causes non-uniform mixing,
 and skewed concentration

Shear-Flow Dispersion

- When velocity is uniform, only advection and turbulent diffusion occurs
- When Velocity is non uniform, an Additional Longitudinal Mixing Process Occurs, that is called **Shear-Flow Dispersion**
- Far below the point of injection, a balance is achieved between shear and transverse mixing, and a one-d analysis is suitable, using a Longitudinal Diffusivity E_L .
- Close to point of injection, 2-d or 3-d analysis should be carried out.
- Magnitude of E_L is 100 to 1000 times greater than turbulent diffusivities (E_x, E_y, E_z)
- Effect of Shear-Flow Dispersion should be taken when 1-d or 2-d analysis is carried out.

Governing Advective-Diffusion Equation

- Governing equation can be derived by applying conservation of mass in a control volume. (Ref to book of Chang, for derivations).
- 3d- AD Equation

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = \frac{\partial}{\partial x} \left(E_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left(E_y \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left(E_z \frac{\partial C}{\partial z} \right) - KC \quad (14.3.5)$$

in which u, v, w = velocity components in the three coordinate directions, x, y, z ; C = concentration in the turbulent flow; E_x, E_y, E_z = nonisotropic (a function of direction), nonhomogeneous (a function of location) turbulent diffusivities in the x, y, z directions; and first-order decay (with coefficient K) is assumed.

Governing Advective-Diffusion Equation

- 2-d AD Equation

Equation (14.3.5) can be applied to two-dimensional problems by averaging in one coordinate direction. For example, if vertical averaging (in the z direction) is accomplished, the two-dimensional, nonisotropic, nonhomogeneous form in cartesian coordinates is^{40,91}

$$\frac{\partial (hC)}{\partial t} + \frac{\partial (u h C)}{\partial x} + \frac{\partial (v h C)}{\partial y} = \frac{\partial}{\partial x} \left(h E'_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left(h E'_y \frac{\partial C}{\partial y} \right) - h K C \quad (14.3.6)$$

For this equation, the concentration and u and v velocities (in the x and y directions) are vertically averaged over the variable depth $h(x, y)$. Thus, E'_x and E'_y include a diffusive mixing component due to shear-flow dispersion. Equation (14.3.6) can be simplified if the depth is constant in space and time, since h can then be eliminated from each term.

- 1-d AD Equation

The one-dimensional form of the advective-dispersion equation commonly applied in streams is⁹¹

$$\frac{\partial C}{\partial t} + \frac{\partial (UC)}{\partial x} = \frac{\partial}{\partial x} \left(A E_L \frac{\partial C}{\partial x} \right) - K C \quad (14.3.7)$$

In Eq. (14.3.7), $U = Q/A$ is the average longitudinal velocity, $A(x)$ is the cross-sectional area at any location, and $E_L(x)$ is the longitudinal dispersion coefficient.

Water Quality Models

EPA

- AQUATOX :
- BASINS: Better Assessment Science Integrating Point and Nonpoint Source
- QUAL2K: River and Stream Water Quality Program
- SWMM: Storm Water Management Model
- WASP: Water Quality Analysis Simulation Program

USGS

- SPARROW : It is Statistical Analysis System SAS based Water Quality Program

HEC RAS and WATER QUALITY

- HEC Ras can perform:
 - detailed temperature analysis, and
 - transport of a limited number of water quality constituents;
 - Algae,
 - Dissolved Oxygen (DO),
 - Carbonaceous Biological Oxygen Demand BOD,
 - Dissolved Orthophosphate,
 - Dissolved Organic Phosphorous,
 - Dissolved Ammonium Nitrate,
 - Dissolved Nitrite Nitrogen,
 - Dissolved Nitrate Nitrogen, and
 - Dissolved Organic Nitrogen,