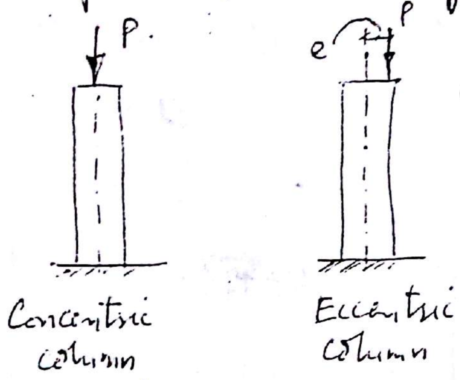
COLUMNS (Lec #1)

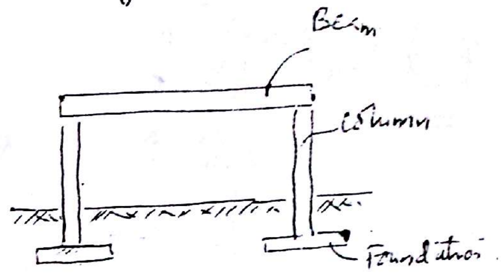
(1) Dec. 25, 2011

Session 2010 - Civil

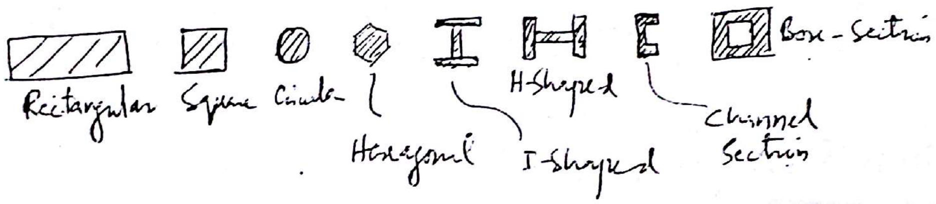
" Column is a structural member subjected to axial compressive loads along its ~~lo~~ longitudinal axis "



e = eccentricity



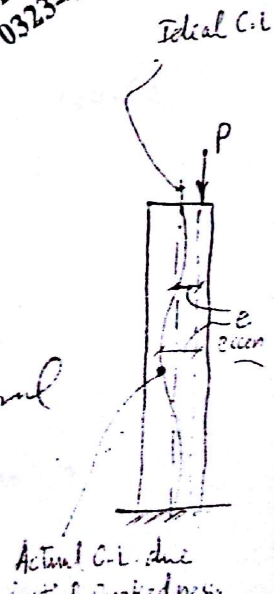
CROSS-SECTIONAL SHAPES & MATERIALS



Material,
- Conc.
- Steel
- Brick
- Wood, etc.

CONCENTRIC COLUMNS (Ideal Column)

A Column which is subjected to axial load perfectly along its longitudinal axis. These columns are very rare.



Eccentric Column

A column which is ~~by~~ subjected to axial load at an eccentricity from its longitudinal axis. These columns have axial, as well as flexural stresses, so they are also called Beam-Columns.

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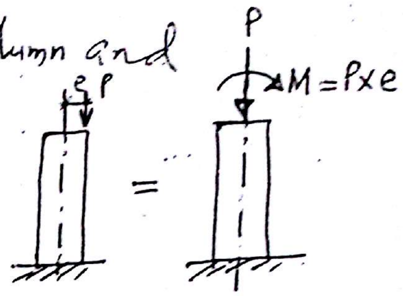
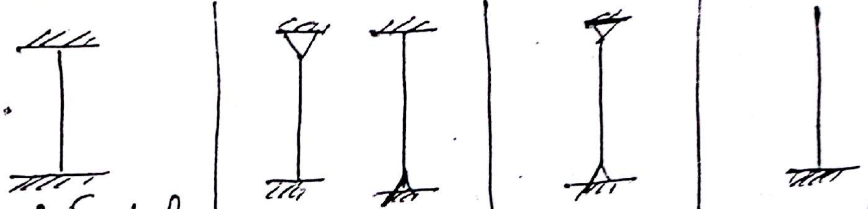
Eccentricity (e) :-

"Distance b/w the actual C.L. of Column and the line of action of force"



End Conditions :-



Fixed Ended Columns | One End pinned one fixed | Hinged Column | One fixed, one free end (Water Tower)

Slenderness Ratio :- (λ)

"It is the ratio of effective length to the least radius of gyration".

$$\text{Slenderness Ratio } (\lambda) = \frac{\text{Effective length } (l_e)}{\text{Min. radius of gyration } (r_{min})}$$

$$\lambda = \frac{l_e}{r_{min}} = \frac{k\,l_u}{r_{min}} \Rightarrow \lambda \propto k$$

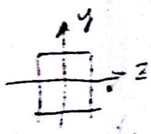
where k = effective length factor (Depends on end Conditions)

l_u = ~~actual length of column~~ . Unsupported length of col.

r = Radius of gyration (mathematical Property) $\left[r = \sqrt{I/A}\right]$

l_e = Effective length

- r_{min} is Smaller of r_y & r_z , $r_y = \sqrt{I_y/A}$, $r_z = \sqrt{I_z/A}$



X-Section of Column.

Buckling

"Sudden Lateral Bending produced due to axial load."

- ~~Chances of buckling are directly related to with the slenderness ratio.~~

Buckling is observed in
~~Bars~~/Columns having relatively large lengths as compared to cross-sectional dimensions. ~~The column suddenly bends at a particular compressive load, this phenomenon is called buckling.~~

- Bending is a slow and gradual process, whereas buckling is a sudden phenomenon, which occurs at critical load. Chances of buckling are directly related ~~to~~ with the slenderness ratio.

Critical Load (Euler's Buckling Load) P_{cr} :-

"The maximum load to which a column can be subjected and still remain straight"

→ This is the load at which buckling just starts

Effective length factor (K) :-

" Ratio of effective length to the unsupported length "

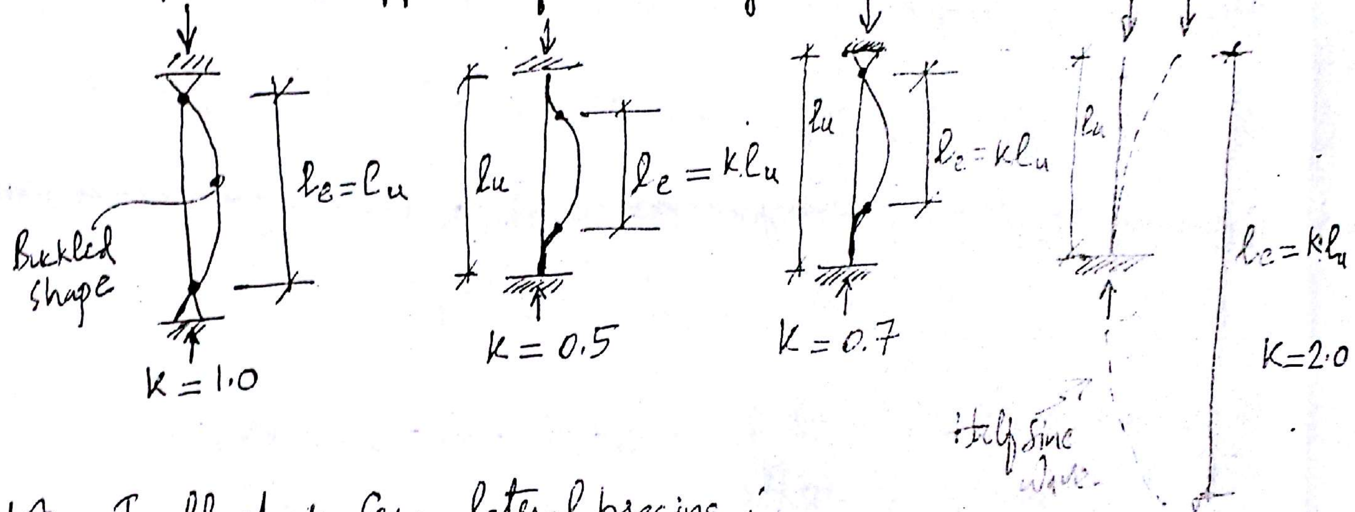
$$K = \frac{l_e}{l_u} \Rightarrow l_e = K l_u$$

Effective length :- (l_e)

(i) The length of column corresponding to one-half sine wave of the buckled shape

OR

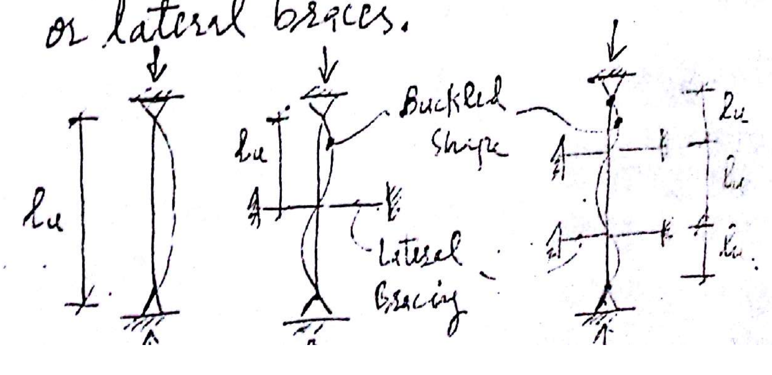
(ii) The length b/w two ~~consective~~ consecutive ~~inflexion~~ inflection points or supports after buckling



Note: In all above cases lateral bracing is only present at the supports.

Unsupported length :- (l_u)

" The length of column b/w two consecutive supports or lateral braces."

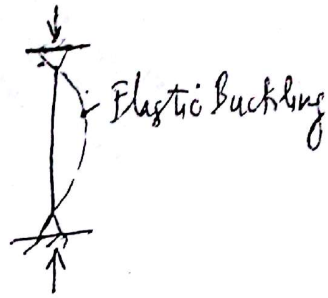


Note :- # different values of unsupported length may exist in different directions.

Types of Columns depending on Buckling Behaviour

Long Column :-

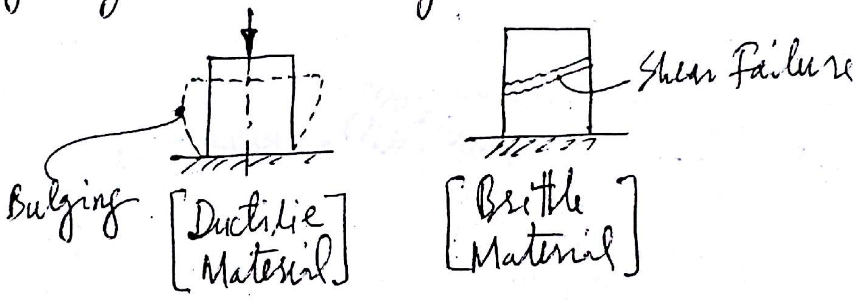
"In long Columns, elastic buckling is produced and the deformations due to this type of buckling are recovered upon removal of the load".



$\boxed{\text{Slenderness Ratio} > 100}$

Short Column :-

" In Short Columns, failure takes place due to yielding & no buckling occurs"



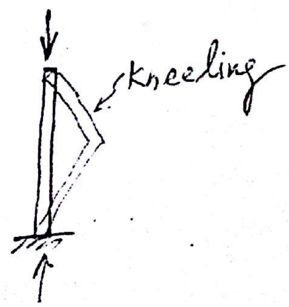
• In practice, very few columns meet this Condition.

$\boxed{\text{Slenderness Ratio} \leq 30}$

Intermediate Column :-

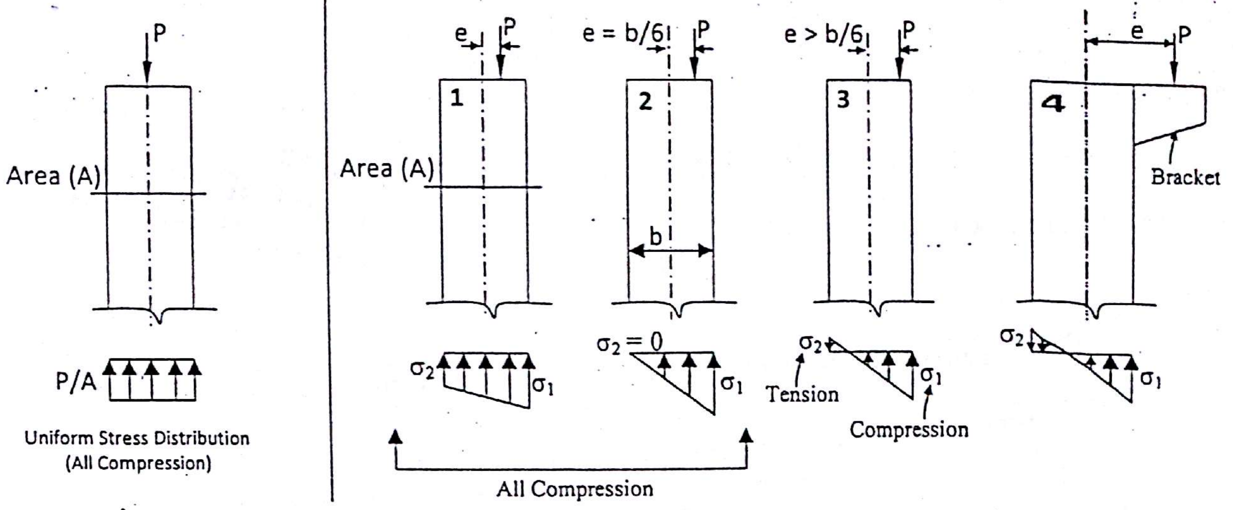
• Intermediate Columns buckle at relatively higher load as compared with ~~load~~ long Columns. The buckling is in-elastic meaning that column does not recover its shape upon removal of load.

Slenderness Ratio = 30 – 100



Stress Distribution in Columns

Concentric Columns | **Eccentric Columns**

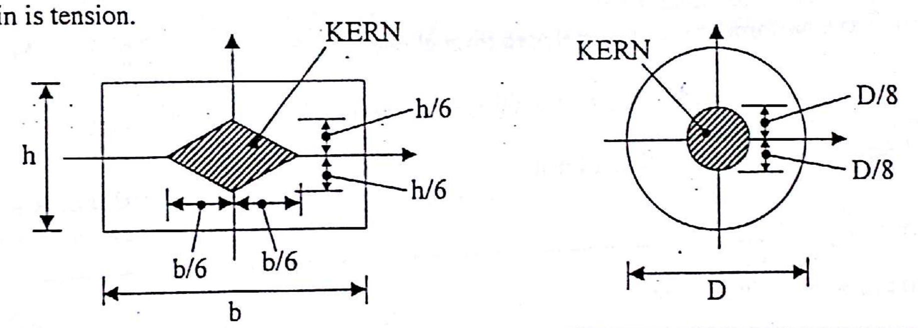


σ_A = Axial Stress = P/A σ_b = Bending Stress = My/I

σ = P/A ± My/I σ_1 = P/A + My/I σ_2 = P/A - My/I

KERN of the Section

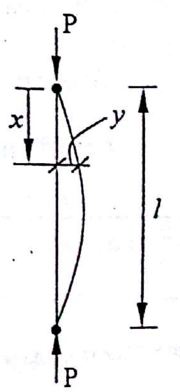
An area in the cross-section within which if the resultant compressive load lies no part of the cross-section in is tension.



Euler's Buckling Theory (1757 A.D.)

Euler analyzed the critical load for long columns (elastic buckling) based on the differential equation of the elastic curve: $EI\frac{d^2y}{dx^2} = M$

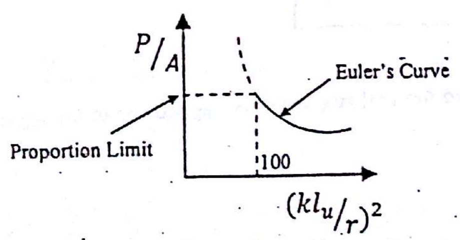
$$\boxed{\text{Euler's Formula: } P_{cr} = \frac{\pi^2 EI}{l^2}}$$



P_{cr} is elastic critical buckling load. This formula was derived only for hinged columns. Generally, columns are not hinged therefore Euler's formula needs to be generalized.

$$P_{cr} = \frac{\pi^2 EI}{l_e^{\,2}} = \frac{\pi^2 E.Ar^2}{(kl_u)^2} = \frac{\pi^2 EA}{(kl_u/r)^2} \Rightarrow P_{cr}/A = \frac{\pi^2 E}{(kl_u/r)^2} \Rightarrow \sigma_{cr} = \frac{\pi^2 E}{(kl_u/r)^2}$$

$\sigma_{cr} = Critical\ Stress$



Critical loads for columns with end conditions other than hinged can be expressed in terms of critical load for hinged columns:

$$P_{cr\,(for\ other\ end\ conditions)} = N.\ P_{cr\,(hinged\ column)}$$

For hinged columns: k = 1.0

$$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 EI}{l_u{}^2}$$

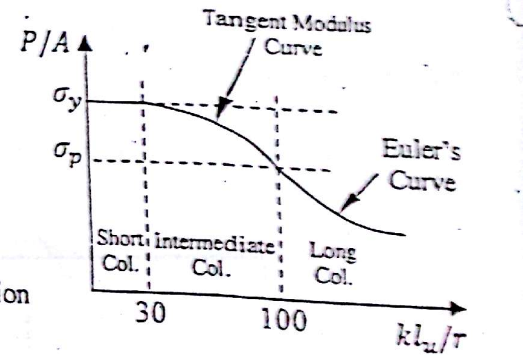
For fixed ended columns: k = 0.5

$$P_{cr} = \frac{\pi^2 EI}{(0.5l_u)^2} = 4.\frac{\pi^2 EI}{l_u{}^2}$$

End Condition	N = Number of times strength of hinged column	Effective length, $l_e = kl_u$
Hinged	1	$l_e = l_u$
Fixed Ended	4	$l_e = 0.5l_u$
One End Fixed, other Hinged	2	$l_e = 0.7l_u$
One End Fixed, other Free	~~0.5~~ 0.25	$l_e = 2.0l_u$

Limitations/Assumptions of Euler's Formula

1. Column must be initially straight
2. Column must be of homogeneous material
3. Column carries perfectly axial load (e=0)
4. Column should have uniform cross-sectional area throughout
5. Self-weight of the column is neglected
6. Column is very long as compared to its cross-sectional dimension
7. Column fails only due to buckling
8. Stress should not exceed the proportional limit



Stress~Slenderness Ratio Curve

σ_y = yield Stress, σ_p = Stress at P.L

Rankine-Jordan Formula (for critical load)

$$P_{cr} = \frac{f_u.A}{1 + a.(^{l_u}/_{r})^2}$$

where

f_u = *ultimate stress* = σ_u

$$Constant,\ a = \frac{f_u.k^2}{\pi^2 E}$$

k = *Effective length factor*

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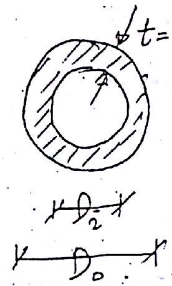
Mechanics of Solids (3rd Term B.Sc Civil & CTE)
(Column Problems)

Problem 3. Find 'Euler's Buckling Load' for a column having outer diameter 250mm and 20mm wall thickness. The length of the column is 10m and it is hinged at both ends.
Take $E = 77{,}000$ MPa, $f_u = 556$ MPa.
~~Also calculate 'Buckling Load' by 'Rankine-Jordon' Formula.~~

Solution.

$t = 20\,mm$



$D_i = 210\,mm$

$D_o = 250\,mm$

$$A = \frac{\pi}{4}(D_o^2 - D_i^2)$$
$$= 14451.32\ mm^2$$

$$I = \frac{\pi}{64}(D_o^4 - D_i^4)$$
$$= 96.28 \times 10^6\ mm^4$$

For circle $I = \frac{\pi d^4}{64}$

$$r = \sqrt{\frac{I}{A}} = 81.62\,mm$$

Euler's Critical Load.

$$P_{cr} = \frac{\pi^2 E I}{(Kl)^2} = \frac{\pi^2 \times 77000 \times 96.28 \times 10^6}{(1 \times 10{,}000)^2} = 731689.04\,N$$
$$= 731.69\ KN$$

Rankine-Jordon Formula

$$P_{cr} = \frac{f_u \cdot A}{1 + a\left(\frac{l}{r}\right)^2}$$

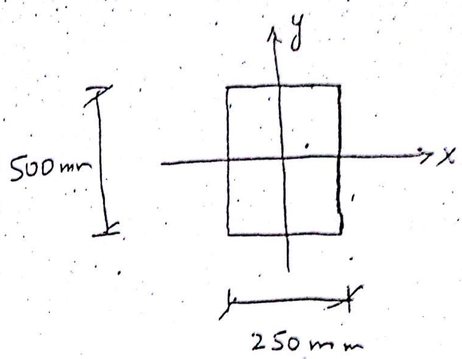
$$a = \frac{f_u \cdot K^2}{\pi^2 \cdot E} = \frac{556 \times 1.0^2}{\pi^2 \times 77000}$$

$$\therefore a = \frac{1}{1366.83}$$

$$= \frac{556 \times 14451.32}{1 + \frac{1}{1366.83}\left(\frac{10{,}000}{81.62}\right)^2}$$

$$= 670569.97\,N \cong 670.57\ KN$$

Problem 02: A small steel bar of rectangular cross section 250 mm × 500 mm is to be used as column with fixed ends.
What minimum length of the column can be used in Euler's Formula if E = 200 GPa and critical stress is 210 MPa.



Solution.

$$A = 125000 \text{ mm}^2.$$

$$I_{max} = I_x = 2.604 \times 10^9 \text{ mm}^4$$

$$I_{min} = I_y = 651.04 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I_{min}}{A}} = 72.17 \text{ mm}$$

$$P_{cr} = \frac{\pi^2 E A}{\left(\frac{Kl}{r}\right)^2}$$

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{\left(\frac{Kl}{r}\right)^2}$$

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{Kl}{r}\right)^2}$$

$$210 = \frac{\pi^2 \times 200 \times 10^3}{\left(\frac{0.5 \times l}{72.17}\right)^2}$$

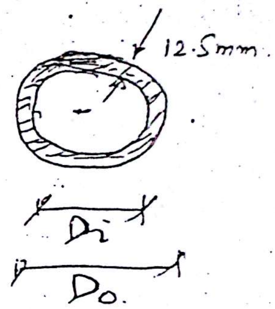
$$l^2 = 195832085.2$$

$$\Rightarrow l = 13994 \text{ mm.}$$

$$\boxed{l \simeq 14 \text{ m}}$$

Problem 03: Calculate the Euler's Critical (Buckling) Load for a hollow cast iron column 12 m long and having 300mm outer diameter and 12.5mm wall thickness fixed at both ends. $E = 18.5\ kN/mm^2$

Also calculate the minimum length for which the Euler's Formula is valid if $\sigma_{cr} = 22\ N/mm^2$.



Solution. $A = \frac{\pi}{4}(300^2 - 275^2) = 11290\ mm^2$

$I = \frac{\pi}{64}(300^4 - 275^4) = 116.87 \times 10^6\ mm^4$

$r = \sqrt{\frac{I}{A}} = 101.74\ mm$

$l = 12000\ mm$

$$P_{cr} = \frac{\pi^2 E I}{(Kl)^2} = \frac{\pi^2 \times 18.5 \times 10^3 \times 116.87 \times 10^6}{(0.5 \times 12000)^2} = 592.75\ kN$$

To find length

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{Kl}{r}\right)^2}$$

$$l^2 = \frac{\pi^2 E r^2}{\sigma_{cr} \cdot K^2} = \frac{\pi^2 \times 18.5 \times 10^3 \times 101.74^2}{22 \times 0.5^2}$$

$\rightarrow l = 18537\ mm$

$\boxed{l = 18.54\ m}$

Problem No. 04: Calculate the size of a square column that is to support a critical load of 1000 KN. Column is 12 m long, $E = 25\ KN/mm^2$.
A factor of safety of 2 is to be used in design. Use Euler's Formula. (Both ends are pinned.)

Solution:

Factor of safety (F.O.S) = 2.0

$\Rightarrow \quad P_{cr} = 1000 \times 2 = 2000\ KN$

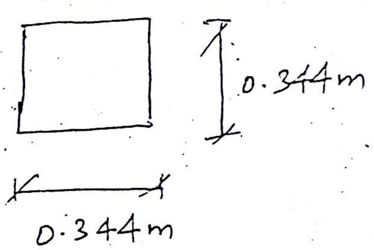
square column $\Rightarrow \quad I = \frac{b^4}{12}$

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2}$$

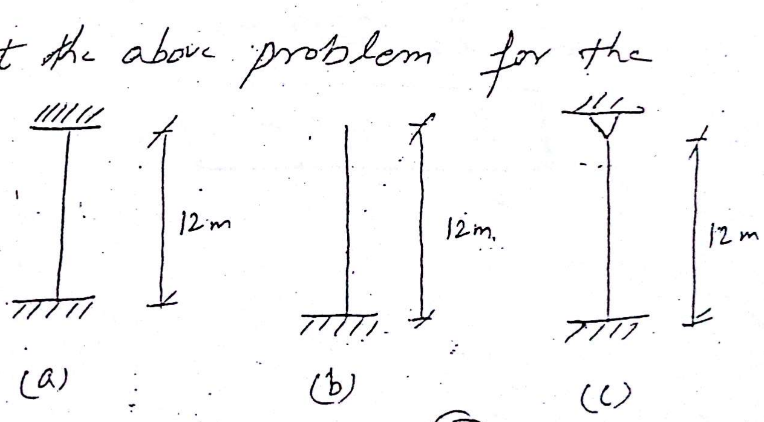
$$2000 = \frac{\pi^2 \times 25 \times \left(\frac{b^4}{12}\right)}{(1.0 \times 12000)^2}$$

$\Rightarrow \quad b = 344\ mm = 0.344\ m$

$\Rightarrow$



Problem No. 05 Repeat the above problem for the following conditions.



Problem No. 06

(a) Compare buckling load of a column fixed at both ends obtained by Eulers & Rankine Jordan Formula for the following condition -

Actual Length, $l = 20$ ft.

Diameter, $d = 8$ inch.

$E = 24\times10^6$ psi, $\sigma_{cr} = 30,000$ psi

$a = \frac{1}{200000}$

(b) For what length of column both the formulae will give same critical load.

Solution

Eulers | Rankine - Jordan

(a)

$$P_{cr} = \frac{\pi^2 (24\times10^6)\times 50.266}{\left(\frac{0.5\times 20\times12}{2}\right)^2}$$

$$P_{cr} = 3307.37 \text{ kips}$$

$$P_{cr} = \frac{30,000\times 50.266}{1+\frac{1}{200000}\left(\frac{20\times12}{2}\right)^2}$$

$$P_{cr} = 1406.7 \text{ kips}$$

(b)

$$\frac{\pi^2(24\times10^6)\times 50.266}{0.0625\times l^2} = \frac{30,000\times 50.266}{1+\left(\frac{1}{200,000}\right)\frac{l^2}{4}}$$

$$1875 l^2 = 236.87\times10^6 + 296.09 l^2$$

$$l = 387.33 \text{ inch} = 32.28 \text{ ft}$$

Problem No. 07 Solve above problem for 30 ft long and 8" x 8" square column having one end fixed and one free.

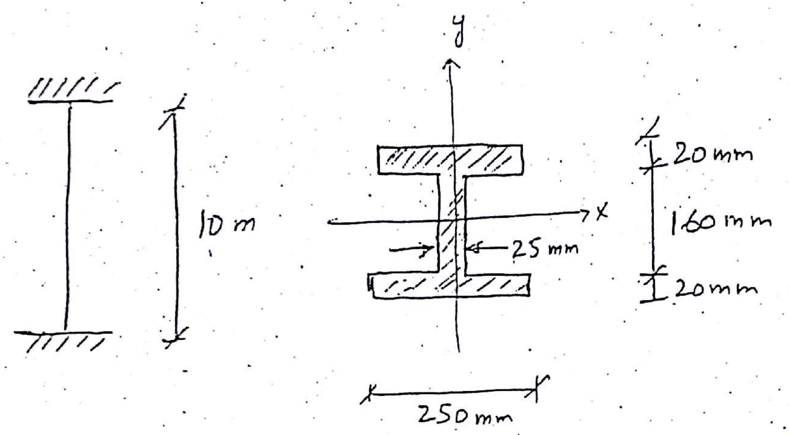
Problem No. 08

For the steel columns given below, comment about

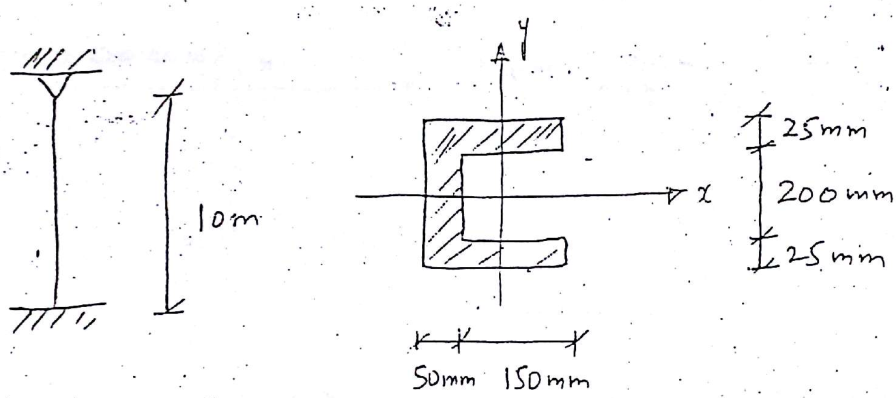
i) Type of column (On the basis of slenderness ratio)

ii) Mode of Failure (Crushing of material, Plastic Buckling or Elastic Buckling).

(a)



(b)



(c)