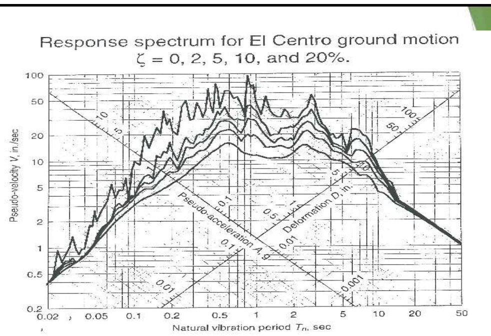
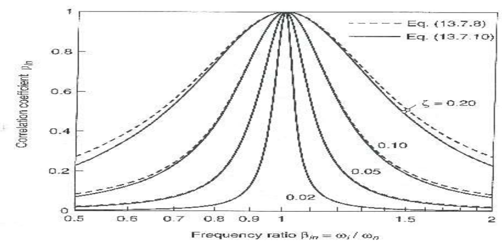
Dynamics of Structures

Dr. Muhammad Irfan-ul-Hassan

Modal Spectral Analysis

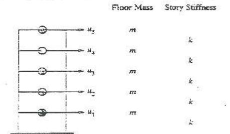


Correlation coefficient



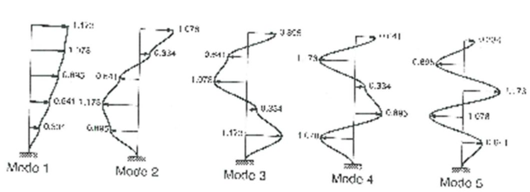
Example: A five story shear building

$m_i = m = 100kips/g$; $k_i = k = 31.54kips/in$; $h_i = h = 12ft$



$$[M] = m \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \; ; \; [K] = k \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix}$$

Natural vibration modes of five-story shear building



The natural frequencies, modes and modal properties are computed as follows:

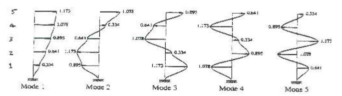
$$\omega_1 = 0.285\sqrt{\frac{k}{m}} \; ; \quad \omega_2 = 0.831\sqrt{\frac{k}{m}} \; ; \quad \omega_3 = 1.310\sqrt{\frac{k}{m}} \; ;$$

$$\omega_4 = 1.682\sqrt{\frac{k}{m}} \; ; \quad \omega_5 = 1.919\sqrt{\frac{k}{m}}$$

$$T_1 = 2.0\text{sec} \; ; \quad T_2 = 0.6852\text{sec} \; ; \quad T_3 = 0.4346\text{sec} \; ;$$

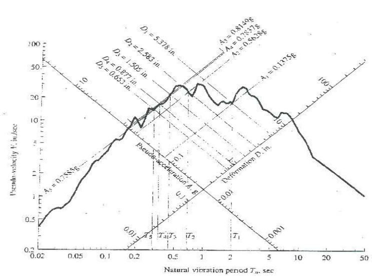
$$T_4 = 0.3383\text{sec} \; ; \quad T_5 = 0.2966\text{sec}$$

modal vectors



$$m_i^* = 1.0$$

$$\Gamma_1 = 1.067 \; ; \quad \Gamma_2 = -0.336 \; ; \quad \Gamma_3 = 0.177 \; ; \quad \Gamma_4 = -0.099 \; ; \quad \Gamma_5 = 0.045$$



Using the response spectrum as shown (assume $\xi_i = 5\%$), the floor displacements can be computed as:

U_{max}

$$\{u\}_i = \Gamma_i \{\phi\}_i D_i$$

$$\{u\}_1 = 1.067 \begin{Bmatrix} 0.334 \\ 0.641 \\ 0.895 \\ 1.078 \\ 1.173 \end{Bmatrix} 5.378 = \begin{Bmatrix} 1.916 \\ 3.677 \\ 5.139 \\ 6.188 \\ 6.731 \end{Bmatrix} \text{in.}$$

1st Mode T=2 sec

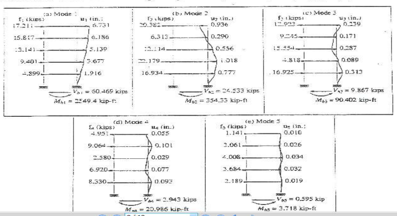
The elastic forces can be computed as:

$$\{f\}_i = \Gamma_i [M] \{\phi\}_i A_i$$

$$\{f\}_1 = 1.067(100/g) \begin{Bmatrix} 0.334 \\ 0.641 \\ 0.895 \\ 1.078 \\ 1.173 \end{Bmatrix} 0.1375g = \begin{Bmatrix} 4.899 \\ 9.401 \\ 13.141 \\ 15.817 \\ 17.211 \end{Bmatrix} \text{kips}$$

1st Mode T=2 sec

The results of the peak displacements and static lateral forces per each mode are shown in the figure. Based on the modal responses, the base shear V_b, base moment M_b, and the displacement u_5 of the top floor per each mode can be calculated as shown in the figure. (NOTE: Alternatively, one can calculate response such as the base shear and base moment using the general equation: $r_i = r_i^{st} S_a(\xi_i, T_i)$.)



Modal combinations: For example, calculation of base shear

- Absolute sum rule:

$$V_{b,max} \leq \sum_{i=1}^{n} |V_{bi,max}|$$
$$= 60.469 + 24.533 + 9.867 + 2.943 + 0.595 = 98.407\text{kips}$$

- Square-Root-Sum-Square (SRSS) rule:

$$V_{max} \approx \sqrt{\sum_{i=1}^{n} (V_{bi,max})^2}$$
$$= \sqrt{60.469^2 + 24.533^2 + 9.867^2 + 2.943^2 + 0.595^2} = 66.066\text{kips}$$

SQUARE MAX
Never square individ
then add

- Complete Quadratic Combination (CQC) rule:

$$V_{max} \approx \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij} V_{bi,max} V_{bj,max}}$$

The calculations of the natural frequency ratio β_{ij} and the correlation coefficients ρ_{ij} are shown as

Natural frequency ratio β_{ij}

Mode (i,j)	j=1	j=2	j=3	j=4	j=5
1	1.000	0.343	0.217	0.169	0.148
2	2.919	1.000	0.634	0.494	0.433
3	4.602	1.576	1.000	0.778	0.683
4	5.911	2.025	1.285	1.000	0.877
5	6.742	2.310	1.465	1.141	1.000

Correlation coefficients ρ_{ij}

Mode (i,j)	j=1	j=2	j=3	j=4	j=5
1	1.000	0.007	0.003	0.002	0.001
2	0.007	1.000	0.044	0.018	0.012
3	0.003	0.044	1.000	0.136	0.062
4	0.002	0.018	0.136	1.000	0.365
5	0.001	0.012	0.062	0.365	1.000

Individual V_b term in CQC rule

Mode (i,j)	j=1	j=2	j=3	j=4	j=5
1	3656.476	10.172	1.615	0.306	0.049
2	10.172	601.844	10.687	1.284	0.178
3	1.615	10.687	97.354	3.943	0.365
4	0.306	1.284	3.943	8.658	0.639
5	0.049	0.178	0.365	0.639	0.354

→ $\sqrt{\Sigma\Sigma}$

Total base shear = 66.507 kips.

Comparison of response history analysis and response spectrum analysis

	V_b (kips)	V_5 (kips)	M_b (kip-ft)	u_5 in
ABSSUM	98.407	56.608	3018.8	7.971
SRSS	66.066	30.074	2575.6	6.800
CQC	66.507	29.338	2572.7	6.793
RHA	73.273	35.217	2593.2	6.847