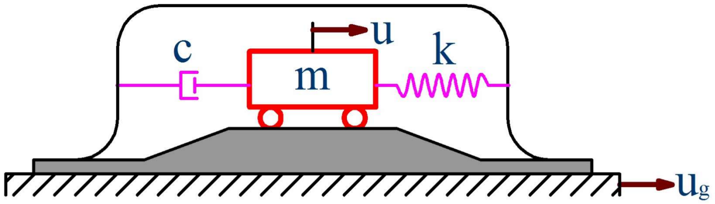
DYNAMICS OF STRUCTURES

Vibration Measuring Instruments

- Vibration measurement is very important in structural engineering, e.g. measurement of ground motion, building motion during earthquake and laboratory testing
- Although today's vibration measuring instruments also called *seismic instruments*, are highly developed and intricate, the underlying principle is simple, i.e. *spring-mass-damper system*
- When subjected to motion, the mass moves and the motion is recorded after suitable *magnifications*
- Different instruments are required for *seismological observation* and earthquake engineering measurements

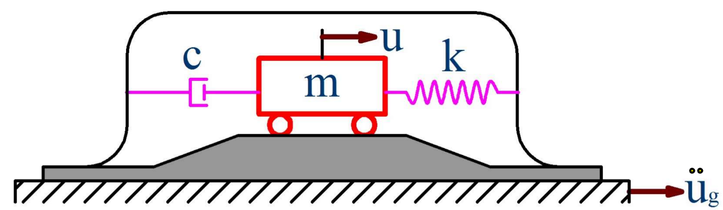


Measurement of Acceleration

- Let the support move with a harmonic acceleration given by:

$$\ddot{u}_g = A \sin \Omega t$$

- The measured displacement and phase angle of the motion of mass are given by: $u(t) = \rho \sin(\Omega t - \phi)$



- Where $\tan\phi = \dfrac{2\zeta\beta}{1-\beta^2}$

$$\rho = \frac{mA}{k}\frac{1}{\sqrt{\left(1-\beta^2\right)^2 + \left(2\zeta\beta\right)^2}} = \frac{A}{\omega^2}\frac{1}{\sqrt{\left(1-\beta^2\right)^2 + \left(2\zeta\beta\right)^2}}$$

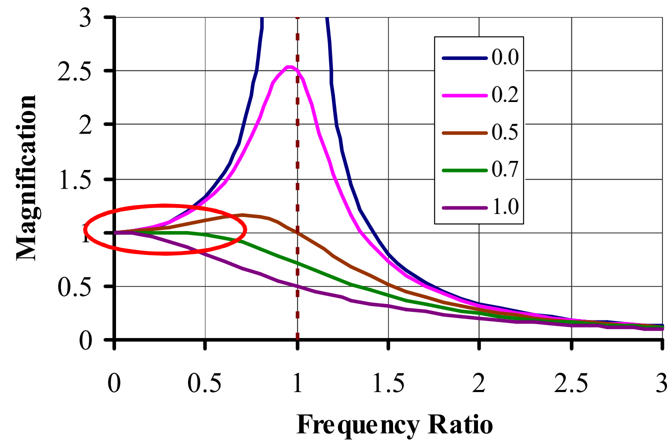
- Now the ratio of the amplitude of measured displacement and the amplitude of ground acceleration is given by:

$$\frac{\rho}{A} = \frac{1}{\omega^2}\frac{1}{\sqrt{\left(1-\beta^2\right)^2 + \left(2\zeta\beta\right)^2}}$$

- The ratio is dependent on the frequency ratio, i.e the frequency of ground acceleration

Measurement of Acceleration (Cont..)

- Since the ground motion due to earthquake is having different frequency contents, the measured response should be independent of frequency of ground motion, i.e. $\rho\omega^2/A$ should not vary with β
- From the table and figure, $\rho\omega^2/A$ remains almost constant for $\zeta = 0.7$ for β varying between 0 and 0.6, if the error is not to exceed 5%
- Since the amplitude ratio (ρ/A) is inversely proportional to frequency of the system (ω), a high instrument frequency will result in low magnification and therefore must be magnified through some other means



β	$\rho\omega^2/A$		Φ/β	
	$\zeta=0$	$\zeta=0.7$	$\zeta=0$	$\zeta=0.7$
0.0	1.00	1.000	0	1.400
0.1	1.01	1.000	0	1.405
0.2	1.04	1.000	0	1.419
0.3	1.10	0.998	0	1.442
0.4	1.19	0.991	0	1.470
0.5	1.33	0.975	0	1.502
0.6	1.56	0.947	0	1.533

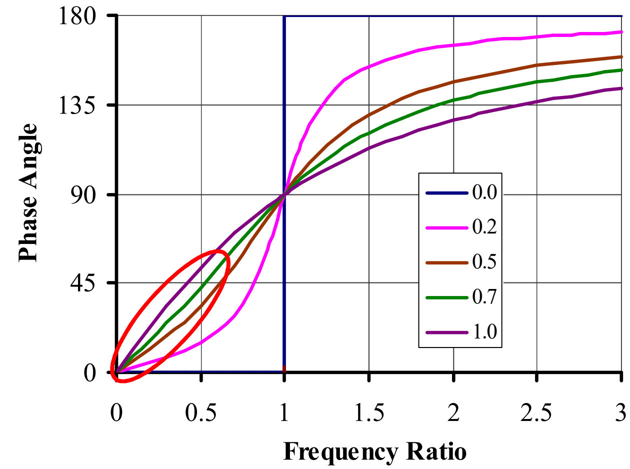
Measurement of Acceleration (Cont..)

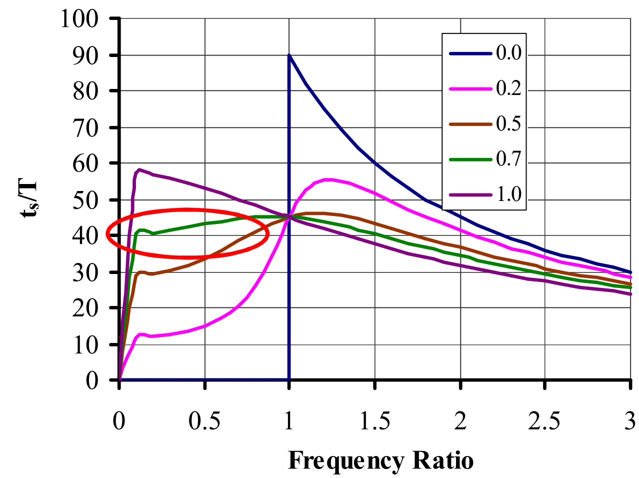
- The time shift between the measure response and ground motion is given by:

$$t_s = \frac{\phi}{\Omega} = \frac{1}{\Omega}\tan^{-1}\frac{2\zeta\beta}{1-\beta^2} = \frac{1}{\beta\omega}\tan^{-1}\frac{2\zeta\beta}{1-\beta^2}$$

$$\frac{t_s}{T} == \frac{1}{2\pi\beta}\tan^{-1}\frac{2\zeta\beta}{1-\beta^2}$$

- When the motion being measured is harmonic, the shift is of no particular significance
- When the motion have several harmonic components as in the case of earthquake, this shift will distort the motion of instrument
- For $\zeta=0.7$ the shift is practically constant, i.e. phase angle is a linear function of β and Ω



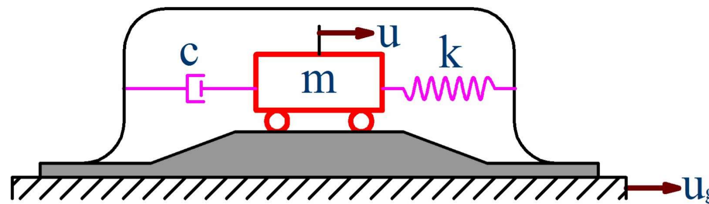


Measurement of Displacement

- Let the support move with a harmonic displacement given by:

$$u_g = G\sin\Omega t$$

- The measured displacement and phase angle of the motion of mass are given by:



$$u(t) = \rho\sin(\Omega t - \phi)$$

- Where $\tan\phi = \dfrac{2\zeta\beta}{1-\beta^2}$

$$\rho = \frac{mG\Omega^2}{k}\frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} = G\frac{\beta^2}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}$$

- Now the ratio of the amplitude of measured displacement and the amplitude of ground displacement is given by:

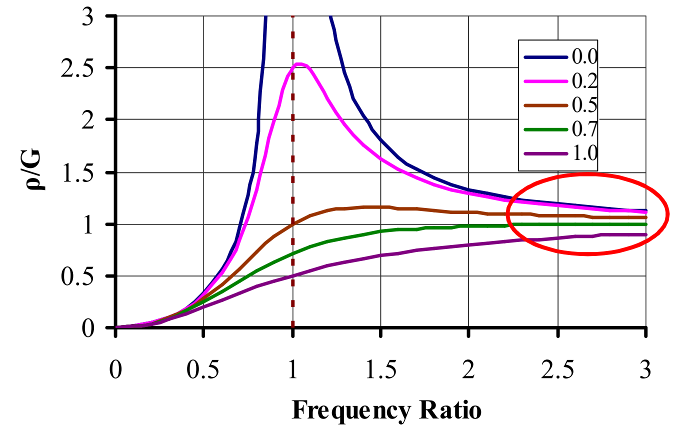
$$\frac{\rho}{G} = \frac{\beta^2}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}$$

- The measured motion is dependent on frequency ratio

Measurement of Displacement (Cont..)

- Since the ground motion due to earthquake is having different frequency contents, the measured response should be independent of frequency of ground motion, i.e. ρ/G should not vary with β
- From the figure, ρ/G remains almost constant for high values of β
- Thus the frequency of displacement measuring instrument shall be very low in comparison with forcing frequency.
- Such instrument is unwieldy because of the heavy mass and soft spring
- For high value of ρ/G is almost unity and phase angle is 180°, for any value of damping therefore:

$$u(t) = -G \sin \Omega t$$



Energy in Viscous Damping

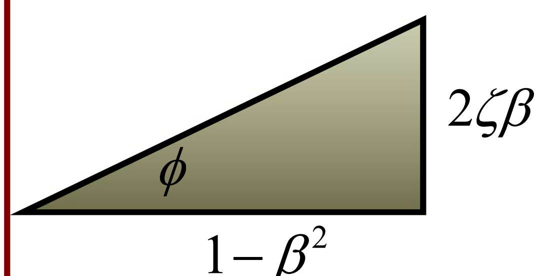
- The energy input per cycle to the system due to the applied force is given by:

$$E_I = \int p(t)du = \int_0^{2\pi/\Omega} (p_o \sin\Omega t)(\dot{u}dt) = \int_0^{2\pi/\Omega} (p_o \sin\Omega t)(\rho\Omega\cos(\Omega t - \phi)dt)$$
$$= \pi\rho p_o \sin\phi$$

- The energy per cycle dissipated in the system is given by:

$$E_D = \int f_D du = \int_0^{2\pi/\Omega} (c\dot{u})(\dot{u}dt) = \int_0^{2\pi/\Omega} c(\rho\Omega\cos(\Omega t - \phi))^2 dt$$
$$= c\pi\Omega\rho^2$$

$$\tan\phi = \frac{2\zeta\beta}{1-\beta^2}$$



- From the figure: $\sin\phi = \dfrac{2\zeta\beta}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} = \dfrac{2\zeta\beta k}{p_o}\rho$

- Therefore: $E_I = \pi\rho p_o\left(\dfrac{2\zeta\beta k}{p_o}\rho\right) = 2\pi\rho^2\zeta\beta k = c\pi\Omega\rho^2$

- Hence: $\boxed{E_I = E_D}$

Energy in Viscous Damping (Cont..)

- The Potential energy per cycle due to spring force is given by:

$$E_S = \int f_S du = \int_0^{2\pi/\Omega} (ku)(\dot{u}dt) = \int_0^{2\pi/\Omega} k(\rho \sin(\Omega t - \phi))(\rho\Omega \cos(\Omega t - \phi))dt = 0$$

- The Kinetic energy per cycle due to inertia force is given by:

$$E_K = \int f_I du = \int_0^{2\pi/\Omega} (m\ddot{u})(\dot{u}dt) = \int_0^{2\pi/\Omega} k(-\rho\Omega^2 \sin(\Omega t - \phi))(\rho\Omega \cos(\Omega t - \phi))dt = 0$$

- For β = 1.0, ($\sin\phi = 1$)

$$E_I = (\pi p_o)\rho \qquad E_D = (2\pi\zeta k)\rho^2$$

- For $E_I = E_D$

$$\pi p_o \rho = (2\pi\zeta k)\rho^2 \Rightarrow \rho = \frac{p_o}{2\zeta k}$$

- Which is the same relation which was already derived using other method

Energy in Viscous Damping (Cont..)

- The spring, inertia and damping forces are given by:

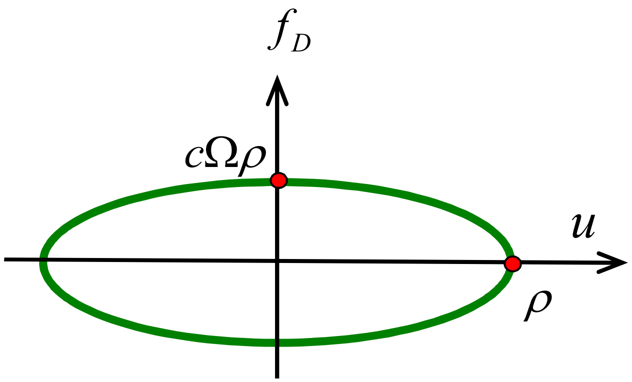
$$f_S = ku$$

$$f_I = -m\Omega^2 u$$

$$f_D = c\dot{u} = c\Omega\rho\cos(\Omega t - \phi)$$

$$f_D = c\Omega\sqrt{\rho^2 - \rho^2 \sin^2 \Omega t} = c\Omega\sqrt{\rho^2 - u^2}$$

$$\Rightarrow \left(\frac{u}{\rho}\right)^2 + \left(\frac{f_D}{c\Omega\rho}\right)^2 = 1.0$$



- Which is the equation of ellipse.
- The area enclosed by the ellipse is equal to dissipated energy
- The f_D-u curve is not a single value function but a loop called *hysteresis loop*

Energy in Viscous Damping (Cont..)

- The total resisting force (elastic + damping) is:

$$f_S + f_D = ku + c\Omega\sqrt{\rho^2 - u^2}$$

- The plot of $f_S + f_D$ is given in the figure which is the same but rotated ellipse