Case 8 – <u>Determination Of Nominal Strength (P_n) For Given Eccentricity ($e < e_b$)</u>

In this case, eccentricity (e) is known and the corresponding failure load (P_n) is to be calculated.

For smaller eccentricities, the failure will be by crushing of concrete in compression and the strength reduction factor (ϕ) will be 0.65 for tied columns.

$e < e_b \quad \Rightarrow \quad P_n > P_{nb} \quad \Rightarrow$ compression failure

A-Exact Solution

Assume that the compression steel is yielding.

Step 1: The tension steel stress f_s is evaluated in terms of 'a' from the strain diagram.

$$f_s = 600\,\frac{\beta_1 d - a}{a}$$

Step 2: The above value of f_s is used in the expression for load and P_n is found in terms of 'a'.

$$P_n = 0.85\,f_c'\,ba + A_s'\,f_y - A_s f_s$$

Step 3: The value of P_n from Step-2 is inserted in the expression for moment to get a cubic equation in terms of 'a'.

$$P_n \times \mathrm{e} = 0.85\,f_c'\,ba\,(d - d'' - a/2) + A_s'\,f_y\,(d - d'' - d) + A_s f_y\,(d'')$$

The resulting cubic equation is solved as under:

i) Simplify the equation such that the right side of equation becomes equal to zero and a polynomial is obtained on the left side.

$$F(a) = 0$$

ii) The derivative of the left side polynomial with respect to 'a' is evaluated.

$$F'(a) = \frac{d}{da} F(a)$$

iii) Trials are made as under until the value of the answer becomes stable.

$$a_{n+1} = a_n - \frac{F(a_n)}{F'(a_n)}$$

1st Trial: Assume $a_o = h/2$ and calculate $F(a_o)$ and $F'(a_o)$.

2nd Trial: $n = 1$, revised value of 'a' $= a_1 = a_o - \dfrac{F(a_o)}{F'(a_o)}$

Calculate $F(a_1)$ and $F'(a_1)$.

The above procedure is repeated until convergence is obtained for the value of 'a'.

Step 4: The yielding of compression steel is checked as under:

$$\varepsilon_s' = 0.003\ \frac{a - \beta_1 d'}{a}$$

$$f_s' = 600\ \frac{a - \beta_1 d'}{a} \quad \leq \quad f_y$$

If the compression steel is not yielding, an expression for f_s' is formulated in terms of 'a' and this value of f_s' is used in Steps 1 and 2 to get a new 3rd order equation in terms of 'a'.

This equation is again solved by the procedure of Step-3 to get the correct value of 'a'.

Step 5: Calculate f_s from the equation in Step-1.

Step 6: Calculate the nominal load from the load equation.

$$P_n = 0.85\, f_c'\, ba + A_s' f_s' - A_s f_s$$

<u>B-Trial Method</u>

This method is not preferable because of very slow convergence of the solution. In some cases, the answer for '*a*' may even oscillate requiring a large number of trials.

Step 1: In the start, the value of '*a*' is assumed greater than $h/2$.

Step 2: The tension steel stress is calculated as follows:

$$f_s = 600\,\frac{\beta_1 d - a}{a}$$

Step 3: The value of P_n is calculated from the moment equation.

$$P_n \times e = 0.85\, f_c'\, ba\,(d - d'' - a/2) + A_s'\, f_y(d - d'' - d') + A_s f_s(d'')$$

Step 4: By using the value of P_n calculated in Step-3, a new value of 'a' is calculated from the load equation.

Step 5: The steps 2 to 4 are repeated until P_n becomes nearly constant.

Case 9 – Determination Of Nominal Strength (P_n) For Given Moment ($P_n < P_{nb}$)

The moment M_n is given and it is known that the failure is by yielding of tension steel. Load P_n is to be found out and the strength reduction factor (ϕ) will be 0.65 to 0.9 for tied columns.

$f_s = f_y$

Assume that the compression steel is yielding, $f_s' = f_y$.

Step 1: Calculate 'a' from the moment equation.

$$M_n = 0.85\, f_c'\, ba\, (d - d'' - a/2) + A_s'\, f_y\, (d - d'' - d) + A_s\, f_y\, (d'')$$

Step 2: The yielding of compression steel is checked as under:

$$\varepsilon_s' = 0.003 \, \frac{a - \beta_1 d'}{a}$$

$$f_s' = 600 \, \frac{a - \beta_1 d'}{a} \leq f_y$$

If the compression steel is not yielding, an expression for f_s' is formulated in terms of '*a*' and this value of f_s' is used in Steps 1. This equation is solved to get the correct value of '*a*'.

Step 3: Calculate the nominal load from the load equation.

$$P_n = 0.85 f_c' \, ba + A_s' f_s' - A_s f_y$$

Step 4: Calculate ε_s and hence evaluate the value of ϕ.

Case 10 – <u>Determination Of Nominal Strength (P_n) For Given Moment ($P_n > P_{nb}$)</u>

The moment M_n is given and it is known that the compression failure will occur by crushing of concrete. Load P_n is to be found out and the strength reduction factor (ϕ) will be 0.65 for tied columns.

$f_s < f_y$

Assume that the compression steel is yielding, $f_s' = f_y$.

Step 1: Find out f_s in terms of 'a'.

$$f_s = 600\,\frac{\beta_1 d - a}{a}$$

Step 2: Calculate 'a' from the moment equation.

$$M_n = 0.85\,f_c'\,ba\,(d - d'' - a/2) + A_s'\,f_y\,(d - d'' - d)$$

$$+ A_s\,600\,\frac{\beta_1 d - a}{a}\,(d'')$$

Step 3: The yielding of compression steel is checked as under:

$$\varepsilon_s' = 0.003\,\frac{a - \beta_1 d'}{a}$$

$$f_s' = 600\,\frac{a-\beta_1 d'}{a} \le f_y$$

If the compression steel is not yielding, an expression for f_s' is formulated in terms of 'a' and the value of 'a' is again calculated.

Step 4: Calculate f_s and f_s' (if compression steel is not yielding).

Step 5: Calculate the nominal load from the load equation.

$$P_n = 0.85\,f_c'\,ba + A_s' f_s' - A_s f_y$$

Step 6: For compression failure, $\phi = 0.65$.

Case 11 – Position Of N.A. (c – Value) Is Given

When the value of depth of neutral axis (c) is known, the analysis becomes straightforward.

Both tension and compression steel stresses can easily be found.

The collapse load is then calculated from the load equation and collapse moment from the moment equation.

This case is always used to plot interaction diagrams.

A number of different values of 'c' may be assumed between 0 and h, and the corresponding points are plotted to get the required curve.

Step 1: Calculate the value of '*a*' from the given value of '*c*'.

Step 2: The tension and compression steel stresses along with the tension steel strain are calculated as follows:

$$f_s = 600 \frac{\beta_1 d - a}{a} \leq f_y$$

$$f_s' = 600 \frac{a - \beta_1 d'}{a} \leq f_y$$

$$\varepsilon_s = 0.003 \frac{\beta_1 d - a}{a}$$

Step 3: Calculate ϕ factor based on the tension steel strain.

Step 4: Calculate the nominal load from the load equation.

$$P_n = 0.85\, f_c'\, ba + A_s' f_s' - A_s f_s$$

-

Step 5: The value of moment capacity M_n is calculated from the moment equation.

$$M_n = P_n \times \mathrm{e} = 0.85\, f_c'\, ba\, (d - d'' - a/2) + A_s' f_y\, (d - d'' - d') + A_s f_s\, (d'')$$

Example 14.1: A reinforced concrete short column has a cross-sectional size of 450 × 300 mm and is loaded as shown in Fig. 14.13. It is reinforced by 6 bars of Grade 300, each having an area of 510 mm^2. $f_c' = 25\ MPa$ and $f_y = 300\ MPa$. Analyze the column for the following conditions:

1) Pure axial load
2) Balanced condition
3) P_u = 1300 kN
4) e = 300 mm

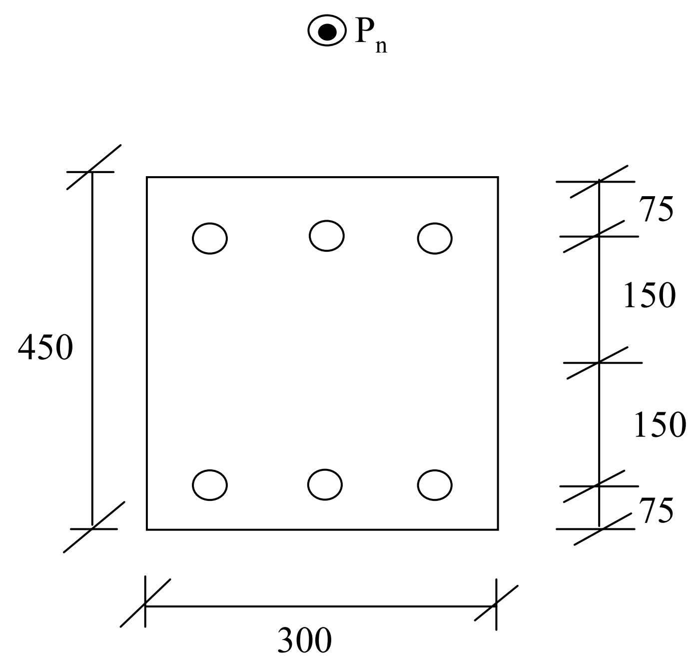

Fig. 14.13.Column Cross-Section For Example 14.1.

Solution:

Location Of Plastic Centroid: By symmetry, the plastic centroid of the given section coincides with the geometric centroid.

1- Pure Axial Load (e=0)

$$
\begin{aligned}
P_{no} &= 0.85\, f_c'\, A_g + A_{st}\,(f_y - 0.85\, f_c') \\
&= [0.85\,(25)(300\times450) \\
&\quad + 6(510)(300-0.85\times25)]\,/\,1000 \\
&= 3721.7\ \text{kN}
\end{aligned}
$$

2- Balanced Condition

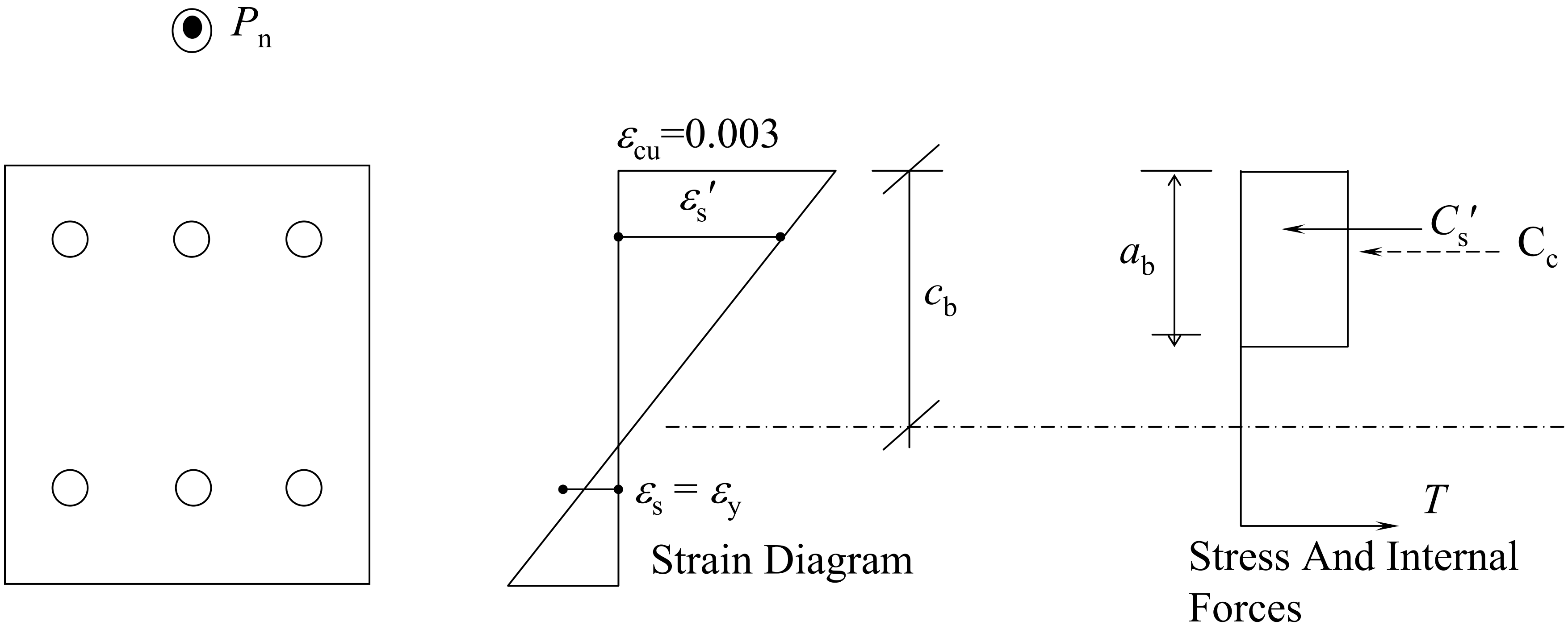

Fig. 14.14. Strain and Stress Diagrams For Case-2.

$$\varepsilon_y = 300 / 200{,}000 = 0.0015$$

$$c_b = \frac{600}{600 + f_y} d = \frac{600}{900} (375) = 250 \text{ mm}$$

$$a_b = \beta_1 c_b = 0.85 (250) = 212.5 \text{ mm}$$

$$\varepsilon_s' = \frac{c_b - 75}{c_b} (0.003) = 0.0021 > \varepsilon_y$$

(Compression steel is yielding)

Internal forces

$$C_c = 0.85 f_c' a_b b = 0.85 (25) (212.5) (300) / 1000$$
$$= 1354.7 \text{ kN}$$

$$
\begin{aligned}
C_s' &= 3\,A_b\,(f_y - 0.85\,f_c') \\
&= 3\,(510)\,(300 - 0.85\times 25)\ \ /\ 1000 \\
&= 426.5 \text{ kN} \\
T &= 3\,A_b f_y \quad = \ 3(510)\,(300)\ /\ 1000 \\
&= 459.0 \text{ kN}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad P_{nb} &= C_c + C_s' - T \\
&= 1354.7 + 426.5 - 459.0 \ = \ 1322.2 \text{ kN} \\
M_{nb} &= \Sigma M_{\text{P.C.}} \\
&= C_c\,(225 - a_b/2) + C_s'\,(150) + T\,(150) \\
&= (1354.7)(0.119) + (426.5)(0.15) \\
&\quad + (459.0)(0.15) \\
&= 294.0 \text{ kN-m}
\end{aligned}
$$

$e_b \quad = \quad M_{nb} / P_{nb} = 222$ mm

3- $\underline{\boldsymbol{P_u} \mathbf{= 1300\ kN}}$

$$P_n \quad = \quad P_u / \phi = 1300 / 0.65 = 2000 \text{ kN}$$

(The strength reduction factor (ϕ) is 0.65 for a tied column in case of compression failure)

$P_n \quad > \quad P_{nb} \quad \Rightarrow$ compression failure is confirmed

$\therefore \quad f_s \quad < \quad f_y$

Assume $f_s' \quad = \quad f_y$

Step 1:

$$f_s = 600 \ \frac{\beta_1 d - a}{a} \quad = \quad 600 \ \frac{0.85 \times 375 - a}{a}$$

Step 2:

$$P_n = 0.85 f_c' \, b \, a + A_s'(f_y - 0.85 f_c') - A_s f_s$$

$$2{,}000{,}000 = (0.85)(25)(300)(a) + (1530)(278.75)$$

$$- \; 1530 \times 600 \; \frac{0.85 \times 375 - a}{a}$$

$$2000\, a = 6.375\, a^2 + 426.488\, a - 292612.5 + 918\, a$$
$$a^2 - 102.83\, a - 45900 = 0$$

$$a = 271.74 \text{ mm}$$
$$c = 271.74 / 0.85 = 319.7 \text{ mm} \quad < \quad d \qquad \text{(OK)}$$

Step 3:

$$f_s' \quad = \quad 600\,\frac{a-\beta_1 d'}{a}$$

$$= \quad 600\,\frac{271.74-0.85\times 75}{271.74}$$

$$= \quad 459.2\ \text{MPa} \quad > \quad f_y$$

$\therefore$ The compression steel is yielding.

Step 4:

$$f_s \quad = \quad 600\,\frac{0.85\times 375-a}{a}$$

$$= \quad 600\,\frac{0.85\times 375-271.74}{271.74} \; = \; 103.80\ \text{MPa}$$

Step 5:

$$M_n = 0.85\, f_c'\, b\, a\, (d - d'' - a/2)$$
$$+ A_s'(f_y - 0.85\, f_c')\, (d - d'' - d') + A_s f_s\, (d'')$$
$$= [(0.85)(25)(300)(271.74)(375 - 150$$
$$- 271.74/2) + (1530)(278.75)(375 - 150$$
$$- 75) + (1530)(103.80)(150)] / 10^6$$
$$= 242.20\ kN\text{-}m$$

4- **<u>*e* = 300 mm</u>**

$e > e_b$ means $P_n < P_{nb}$

$\Rightarrow$ tension failure

$\therefore\ f_s = f_y$

Assume $f_s' = f_y$ (to be checked later)

Step 1: The nominal load capacity P_n is calculated in terms of 'a' as follows:

$$
\begin{aligned}
P_n &= 0.85\, f_c{}'\, \mathrm{ba} + A_s{}'(f_y - 0.85\, f_c{}') - A_s f_y \\
&= (0.85)(25)(300)(a) + (1530)(278.75) \\
&\quad - (1530)(300) \\
&= 6375\, a - 32513
\end{aligned}
$$

Step 2: The value of P_n is used in the expression for moment and the value of 'a' is calculated by solving the resulting equation in terms of 'a'.

$$
\begin{aligned}
P_n \times e = {} & 0.85\, f_c{}'\, ba\,(d - d'' - a/2) \\
& + A_s{}'(f_y - 0.85\, f_c{}')\,(d - d'' - d') + A_s f_y\, d''
\end{aligned}
$$

$$(6375\,a - 32513)(300) \;=\; (6375)(a)[375 - 150 - a/2] + (1530)(278.75)[375 - 150 - 75\,] + (1530)(300)(150)$$

$$a^2 + 150\,a - 44730 \;=\; 0 \qquad a \;=\; 149.4 \text{ mm}$$

Step 3:

$$\varepsilon_s' \;=\; 0.003\,\frac{a-\beta_1 d'}{a}$$

$$=\; 0.003\,\frac{149.4-0.85\times 75}{149.4} \;=\; 0.00172$$

$$> \;\varepsilon_y \qquad \textit{(OK)}$$

Step 4: Calculate load from the load equation.

$$P_n = 0.85\, f_c'\, ba + A_s' f_y - A_s f_y$$

$$= [(6375)(149.4) + (1530)(278.75) - (1530)(300)] / 1000$$

$$= 919.9 \text{ kN}$$

$$M_n = P_n \times e$$

$$= 919.9 \times 300 / 1000 = 275.97 \text{ kN-m}$$

Step 5:

$$\varepsilon_s = 0.003\, \frac{\beta_1 d - a}{a}$$

$$= 0.003\, \frac{0.85 \times 375 - 149.4}{149.4} = 0.0034$$

$$\phi = 0.65 + \frac{0.25}{0.005 - \varepsilon_y}(\varepsilon_t - \varepsilon_y)$$

$$= 0.65 + \frac{0.25}{0.005 - 0.0015}(0.0034 - 0.0015) = 0.79$$

Example 14.2: A reinforced concrete short column has a cross-sectional size of 450 × 300 mm and is loaded as shown in Fig. 14.15. It is reinforced by 6 bars of Grade 300, each having an area of 510 mm^2. $f_c' = 25$ *MPa and* $f_y = 300$ *MPa.* Construct an approximate P-M interaction diagram.

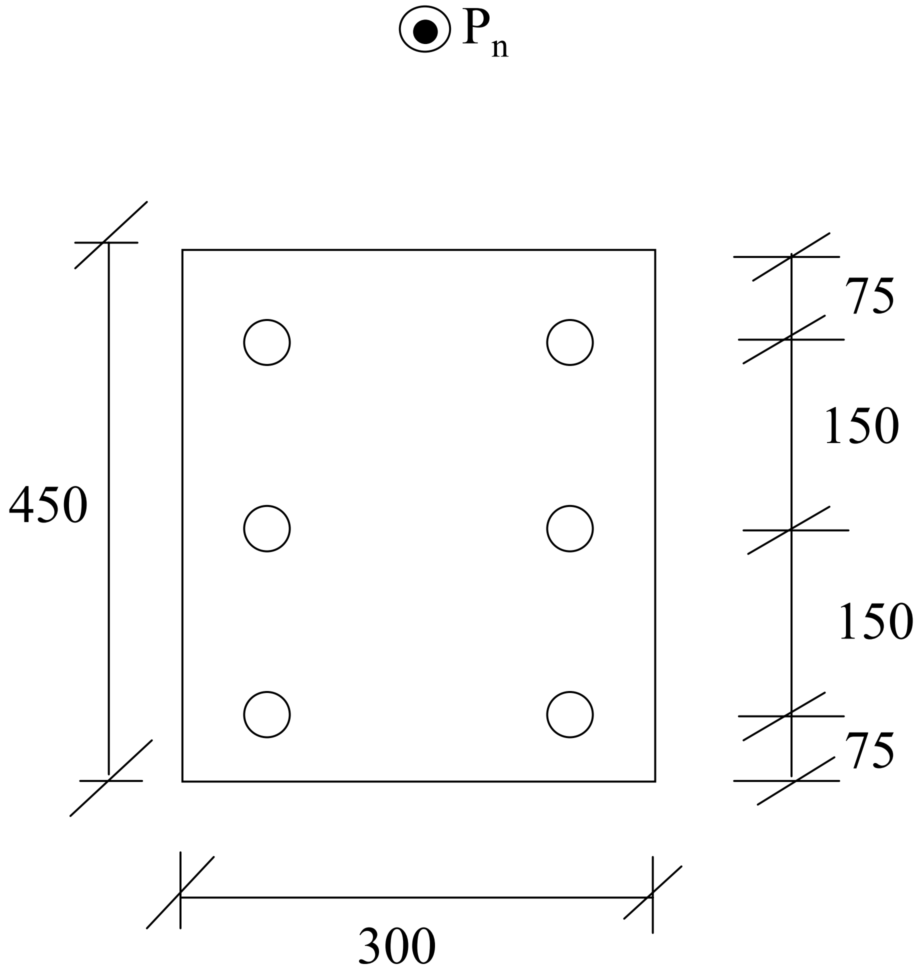

Fig. 14.15.Column Cross-Section For Example 14.2.

Solution:

Location Of Plastic Centroid: By symmetry, the plastic centroid of the given section coincides with the geometric centroid.

Nominal Capacities:

1- Pure Axial Load (e=0)

$$
\begin{aligned}
P_{no} &= 0.85 f_c' A_g + A_{st} (f_y - 0.85 f_c') \\
&= [0.85 (25)(300\times450) + 6(510)(300 \\
&\quad -0.85\times25)] / 1000 \\
&= 3721.7 \text{ kN}
\end{aligned}
$$

2- ***Balanced Condition***

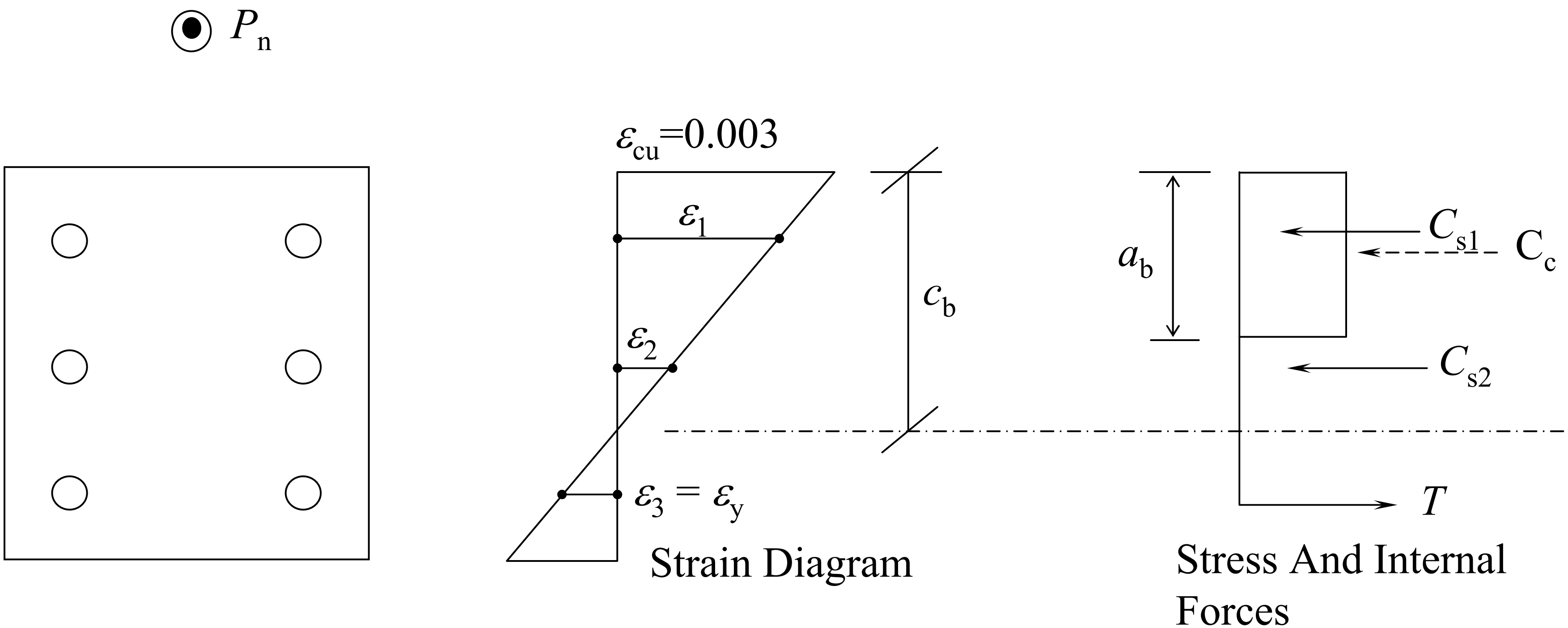

Fig. 14.16. Strain and Stress Diagrams For Case-2.

$$\varepsilon_{\mathrm{y}} = 300 / 200{,}000 = 0.0015$$

$$c_{\mathrm{b}} = \frac{600}{600 + f_y} d = \frac{600}{900} (375)$$

$$= 250 \text{ mm} > \frac{h}{2} = 225 \text{ mm}$$

This means that the central layer of steel is in compression.

$$a_{\mathrm{b}} = \beta_1 c_{\mathrm{b}} = 0.85 (250) = 212.5 \text{ mm}$$

$$\varepsilon_1 = \frac{c_b - 75}{c_b} (0.003) = 0.0021$$

$$> \varepsilon_{\mathrm{y}} \quad \text{(Compression steel is yielding)}$$

$$\varepsilon_2 = \frac{c_b - 225}{c_b}(0.003) = 0.0003$$

Internal forces

$$C_c = 0.85\, f_c'\, a_b\, b$$
$$= 0.85\,(25)\,(212.5)\,(300)\,/\,1000$$
$$= 1354.7 \text{ kN}$$
$$C_{s1} = 2\, A_b\,(f_y - 0.85\, f_c')$$
$$= 2\,(510)\,(300 - 0.85\times 25)\ /\,1000$$
$$= 284.3 \text{ kN}$$
$$C_{s2} = 2\, A_b\, \varepsilon_2\, E_s$$
$$= 2\,(510)\,(0.0003)\,(200{,}000)\,/\,1000 = 61.2\ kN$$
$$T = 2\, A_b f_y = 2(510)\,(300)\,/\,1000 = 306.0\ kN$$

$$\therefore\ P_{nb} = C_c + C_{s1} + C_{s2} - T$$
$$= 1354.7 + 284.3 + 61.2 - 306.0$$
$$= 1394.2 \text{ kN}$$
$$M_{nb} = \Sigma M_{P.C.}$$
$$= C_c\,(225 - a_b/2) + C_{s1}\,(150) + C_{s2}\,(0) + T\,(150)$$
$$= (1354.7)(0.119) + (284.3)(0.15) + 0 + (306.0)(0.15)$$
$$= 249.8 \text{ kN-m}$$

3- <u>At Failure Having ε_s = 0.005</u>

$$\varepsilon_s = 0.005$$
$$c = 0.375\,d = 0.375(375) = 140.6\ mm$$

$$a = 0.85\times140.6 = 119.5 \text{ mm}$$

$$c < h/2$$

$\therefore$ Central layer of steel is in tension

$$\varepsilon_{s1} = (0.003)\,\frac{c-75}{c} = \mathit{0.0014} < \varepsilon_{\mathrm{y}}$$

$$f_{s1} = \varepsilon_{s1}\times E_{\mathrm{s}} = \mathit{280\ MPa}$$

$$\varepsilon_{2} = (0.003)\,\frac{225-c}{c} = 0.0018$$

$\therefore$ Yielding in tension

$$f_{s2} = 300 \text{ MPa}$$

$$P_n = 0.85 f_c' ba + A_{s1} (f_{s1} - 0.85 f_c') - A_{s2} \times f_{s2} - A_{s3} \times f_y$$

$$= [0.85\times25\times300\times119.5 + 1020\times(280 - 0.85\times25) - 1020\times300 - 1020\times300]/1000$$

$$= 413.7 \text{ kN}$$

$$M_n = 0.85 f_c' ba (d - d'' - a/2) + A_{s1} f_{s1} (d - d'' - d') + A_{s2} f_{s2} (0) + A_{s3} f_y (d'')$$

$$= [761812.5 \times (375 - 150 - 119.5/2) + 263925 \times (375 - 150 - 75) + 0 + 306000 (150)] / 10^6$$

$$= 211.4 \; kN\text{-}m$$

4- Pure Flexure

In this case, no axial load is applied.

The load equation reduces to the condition that the internal tension must be equal to the internal compression.

The strain and internal force diagrams are shown in Fig. 14.17, where the location of the N.A. is assumed to be between the top and center layers of steel.

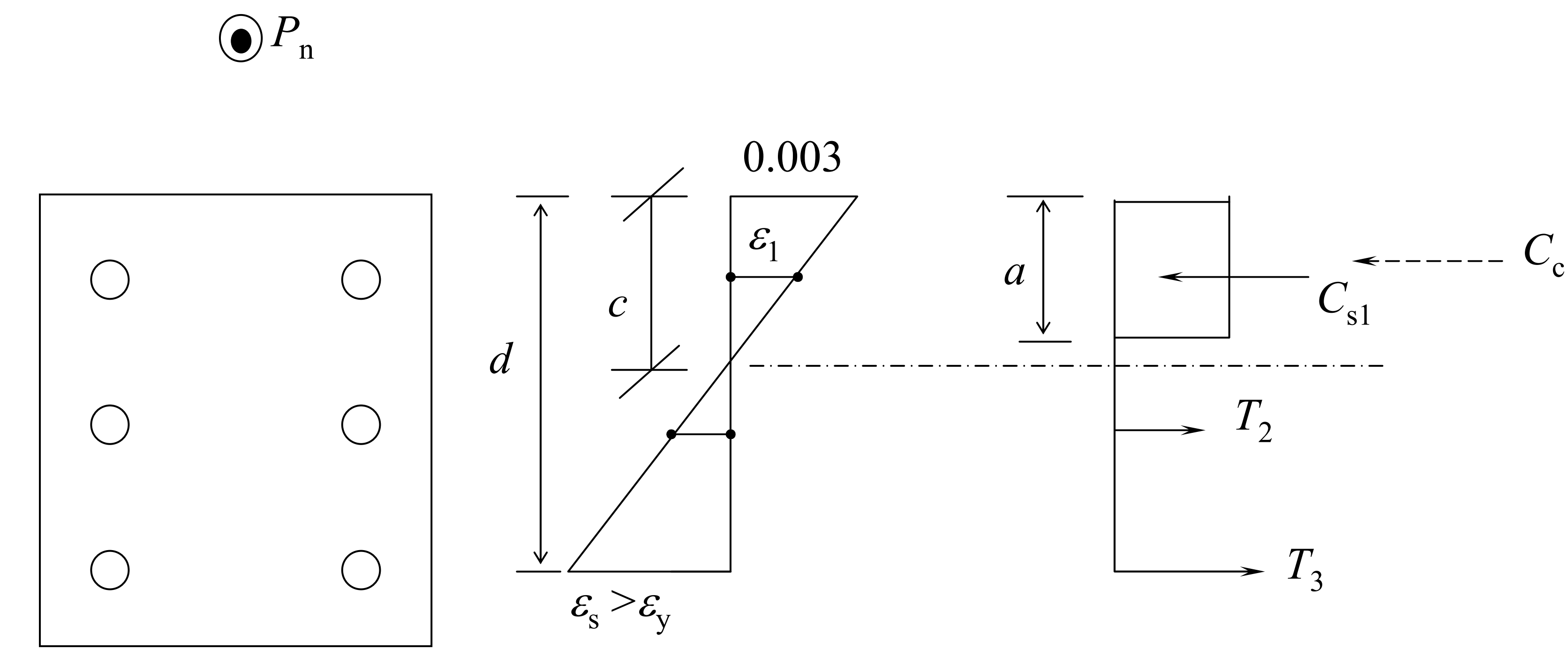

Fig. 14.17. Strain and Stress Diagrams For Case-4.

$$\varepsilon_1 = \frac{c-75}{c}(0.003)$$

$$\varepsilon_2 = \frac{225-c}{c}(0.003)$$

$$C_{s1} = 1020 \times 0.003 \times \frac{c-75}{c} \times 200{,}000$$

$$- 1020 \times 0.85 \times 25$$

$$= 612{,}000 \; \frac{c-75}{c} - 21{,}675$$

$$C_c = 0.85\, f_c' \,(\beta_1 \, c)\, b$$

$$= 0.85 \times 25 \times 0.85 \times c \times 300 \quad = \quad 5419\, c$$

$$T_2 = 2\, A_b\, \varepsilon_1\, E_s$$

$$= 2\, A_b f_y = 1020 \times 300 = 306{,}000 \text{ N}$$

(if yielded)

$$T_3 = 2\, A_b f_y = 1020 \times 300 = 306{,}000\; N$$

$$\Sigma C \quad = \quad \Sigma T \quad \Rightarrow$$

$$612{,}000 \, \frac{c-75}{c} - 21{,}675 + 5419\, c \; = \; 306{,}000 + 306{,}000$$

$$612{,}000\, c - 45{,}900{,}000 - 21{,}675\, c + 5419\, c^2 \; = \; 612{,}000$$

$$5419\, c^2 - 21{,}675\, c - 45{,}900{,}000 \; = \; 0$$

$$c \quad = \; 94.055 \text{ mm}$$

$\therefore$ *N.A. lies in-between steel-1 and steel-2*

$$\varepsilon_1 \quad = \quad \frac{c-75}{c}(0.003) \; = \; 0.00061$$

$$\varepsilon_2 \quad = \quad \frac{225-c}{c}(0.003) \; = \; 0.00418$$

$\therefore$ Steel No. 2 is yielding

$$C_{s1} = 102.314\ kN$$
$$C_c = 509.686\ kN$$
$$P_n = C_{s1} + C_c - T_2 - T_3$$
$$= 509.686 + 102.314 - 2\times306 = 0 \qquad (\boldsymbol{O.K.})$$

$$M_{no} = C_{s1}(225-75)/1000 + C_c \left(225 - \frac{0.85\times94.055}{2}\right)$$
$$/1000 + T_2\,(0) + T_3\,(150)/1000$$
$$= 155.6 \text{ kN-m}$$

<u>Ultimate Capacities</u>

Case – I: $\phi P_{no} = 0.65\,(3721.7) = 2419\ kN$

(without additional factor of safety)

$$(0.80)\ \phi P_{no} = 1935 \text{ kN}$$

Case – II: $\phi M_n = 0.65(249.8) = 162.4\ kN\text{-}m$;

$\phi P_n = 0.65(1394.2) = 906\ kN$

Case – III: $\phi M_n = 0.90(214.6) = 193.1\ kN\text{-}m$;

$\phi P_n = 0.90(435.4) = 392\ kN$

Case – VI: $\phi M_n = 0.90(155.6) = 140.0\ kN\text{-}m$;

$\phi P_n = 0\ kN$

The approximate nominal and design interaction curves are obtained by plotting these points on scale such as in Fig. 14.18.

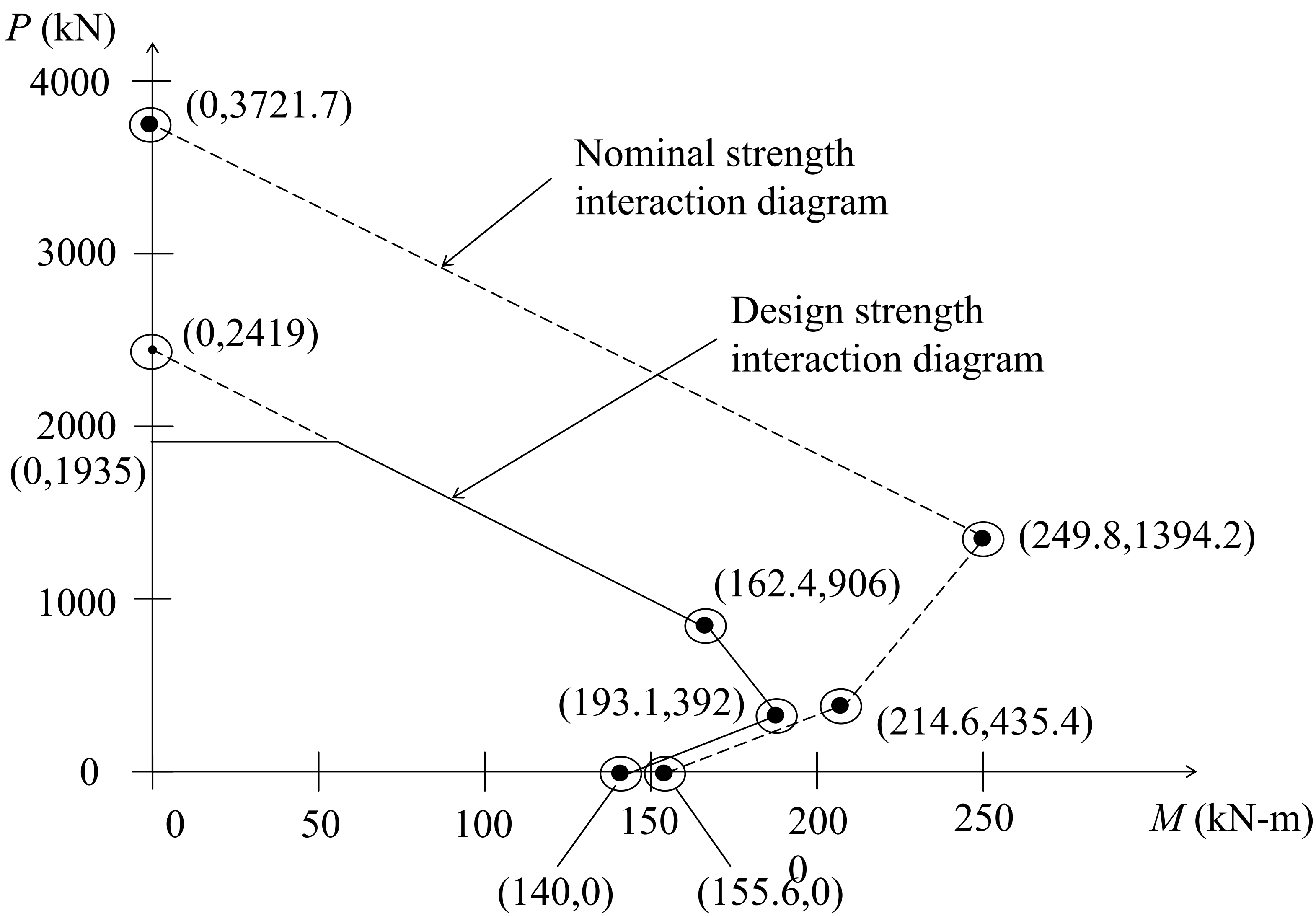

Fig. 14.18. Approximate Interaction Curves For Example 14.2.

DESIGN OF RECTANGULAR SHORT COLUMNS HAVING UNIAXIAL ECCENTRICITY USING FORMULAS

Step 1: First investigate the balanced condition.

$A_s = A_s'$ Assume that the compression steel is yielding for the balanced condition ($f_s' = f_y$).

Calculate a_b and check for yielding of the compression steel.

$$a_b = \beta_1 d \frac{600}{600 + f_y} \qquad \text{(I)}$$

$$\varepsilon_s' = 0.003 \frac{a_b - \beta_1 d'}{a_b} \qquad \text{(II)}$$

If $\varepsilon_s' \geq \varepsilon_y$, the compression steel will be yielding.

Step 2: Calculate $P_{nb} = 0.85 f_c' \, b \, a_b$ (If both steels are yielding and compression steel stress is assumed to be f_y in place of the actual value equal to $f_y - 0.85 f_c'$)

$$(P_{ub})_{design} = P_b = \phi P_{nb} = 0.65 \times 0.85 f_c' \, b \, a_b \qquad \text{(III)}$$

Case I: $P_u < P_b$: Tension Failure

It is known that the tension steel is yielding, $f_s = f_y$.

Step 3: Find approximate value of ϕ.

$$\phi = 0.9 - \frac{P_u}{4P_{nb}} \qquad \text{(VI)}$$

Step 4: Assume the compression steel to be yielding and calculate 'a' from the following equation.

$$\frac{P_u}{\phi} = 0.85 f_c' \, b \, a \qquad \text{(V)}$$

Step 5: Check the yielding of compression steel.

$$f_s' = 600 \, \frac{a - \beta_1 d'}{a} \qquad \text{(VI)}$$

If $f_s' \geq f_y$, the compression steel is yielding $\Rightarrow$ $f_s' = f_y$.

If $f_s' < f_y$, find '*a*' by using procedure of Case II.

Step 6: The value of ϕ may be refined by using the following expressions:

$$\varepsilon_s = 0.003\,\frac{\beta_1 d - a}{a} \qquad \text{(VII)}$$

$$\phi = 0.65 + \frac{0.25}{0.005 - \varepsilon_y}(\varepsilon_t - \varepsilon_y) \qquad \text{(VIII)}$$

Steps 4 and 5 may be revised in case the value of ϕ is significantly different.

Step 7: Taking moments about the plastic centroid, calculate A_s and A_s'.

$$\frac{P_u . e}{\phi} = 0.85\, f_c'\, b\, a\, (d - d'' - a/2) + A_s'\, f_y\, (d - d'' - d') + A_s f_y\, d''$$

For symmetrical sections having same steel and covers on both the sides,

$$A_s = A_s' = A_{st} / 2$$

$$d - d'' = h/2$$

$$\textit{and} \quad d'' = h/2 - d'$$

$$\frac{P_u . e}{\phi} = 0.85 f_c' \, b \, a \left(\frac{h}{2} - \frac{a}{2} \right) + A_{st} f_y \left(\frac{h}{2} - d' \right) \qquad \text{(IX)}$$

Case II: P_u < P_b : Compression Failure

Exact calculations are lengthy for compression failure as compared with tension failure.

However, the compression steel will nearly always be yielding in such cases.

The factor ϕ is always 0.65 for tied columns for compression failure.

Step 3: From the strain diagram, calculate f_s in terms of 'a'.

$$f_s = 600 \frac{\beta_1 d - a}{a} \qquad \text{(X)}$$

Step 4: Calculate A_{st} in terms of 'a' from the load equation.

$$\frac{P_u}{\phi} = 0.85 f_c' \, b \, a + A_s' f_y - A_s f_s$$

$$\frac{P_u}{\phi} = 0.85 f_c' \, b \, a + \frac{A_{st}}{2} f_y - \frac{A_{st}}{2} \times 600 \frac{\beta_1 d - a}{a}$$

(XI)

$$A_{\mathrm{st}} = f_1(a) \qquad \text{(XII)}$$

Step 5: Calculate A_{st} in terms of 'a' from the moment equation.

$$\frac{P_u \, .e}{\phi} = 0.85 \, f_{\mathrm{c}}' \, ba\left(\frac{h}{2} - a/2\right) + \frac{A_{st}}{2} f_{\mathrm{y}} \left(\frac{h}{2} - d'\right)$$

$$+ \quad \frac{A_{st}}{2} \; 600 \; \frac{\beta_1 d - a}{a} \left(\frac{h}{2} - d'\right) \qquad \text{(XIII)}$$

$$A_{st} = f_2(a) \qquad \text{(XIV)}$$

Step 6: Equate A_{st} calculated in steps 4 and 5 to form a cubic equation in terms of 'a'. Solve this equation to calculate the value of 'a'.

$$f_1(a) = f_2(a) \tag{XV}$$

Step 7: Calculate A_{st} from Eq. XII or XIV.