

# VIRTUAL WORK METHOD

1. A suitable yield line pattern is assumed according to the given loads, dimensions and edge conditions.
2. The slab is given a unit virtual displacement at the yield line joining the two plate segments to apply the principle of virtual work.
3. Due to the above displacement, rotation ( $\theta$ ) is calculated at each hinge. The rotation at each yield line is calculated by considering the geometry of the section taken perpendicular to the yield line.

4. Rotation multiplied with the constant moment at the hinge gives the internal work.

$$W_i = \Sigma \theta m_p$$

5. External work is calculated as the product of load and displacement.

$$W_e = \Sigma F d$$

6. External work done is equated to the internal work done to solve for the unknown. Differentiation may be used to minimize the collapse load.

**Example 13.2:** Solve slab system of Example 13.1 using the virtual work method.

**Data:** A one way, uniformly loaded and continuous slab panel of 4m span having a positive flexural capacity of 30 kN-m/m and negative flexural capacities of 30 kN-m/m and 40 kN-m/m at the left and the right supports, respectively. Calculate the ultimate load capacity of the slab.

**Solution:**

Give a unit virtual displacement to yield line-B for the slab system shown in Fig. 13.14.

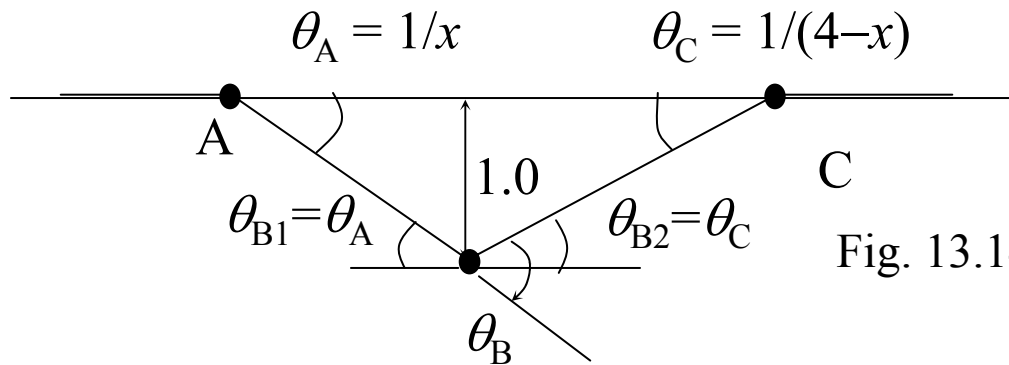
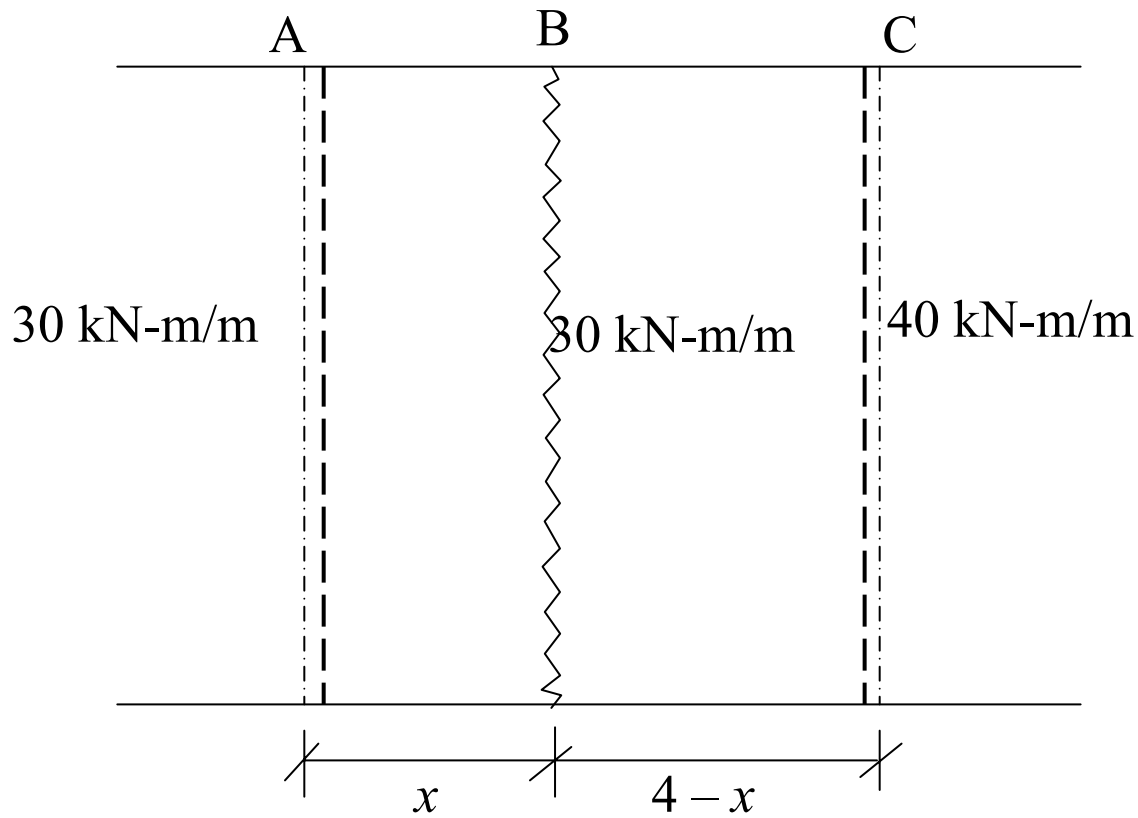


Fig. 13.14.

Solution of One-Way  
Continuous Slab by  
Virtual Work Method.

$$\begin{aligned}\theta_B &= \theta_{B1} + \theta_{B2} \\ &= \frac{1}{x} + \frac{1}{4-x} = \frac{4}{x(4-x)}\end{aligned}$$

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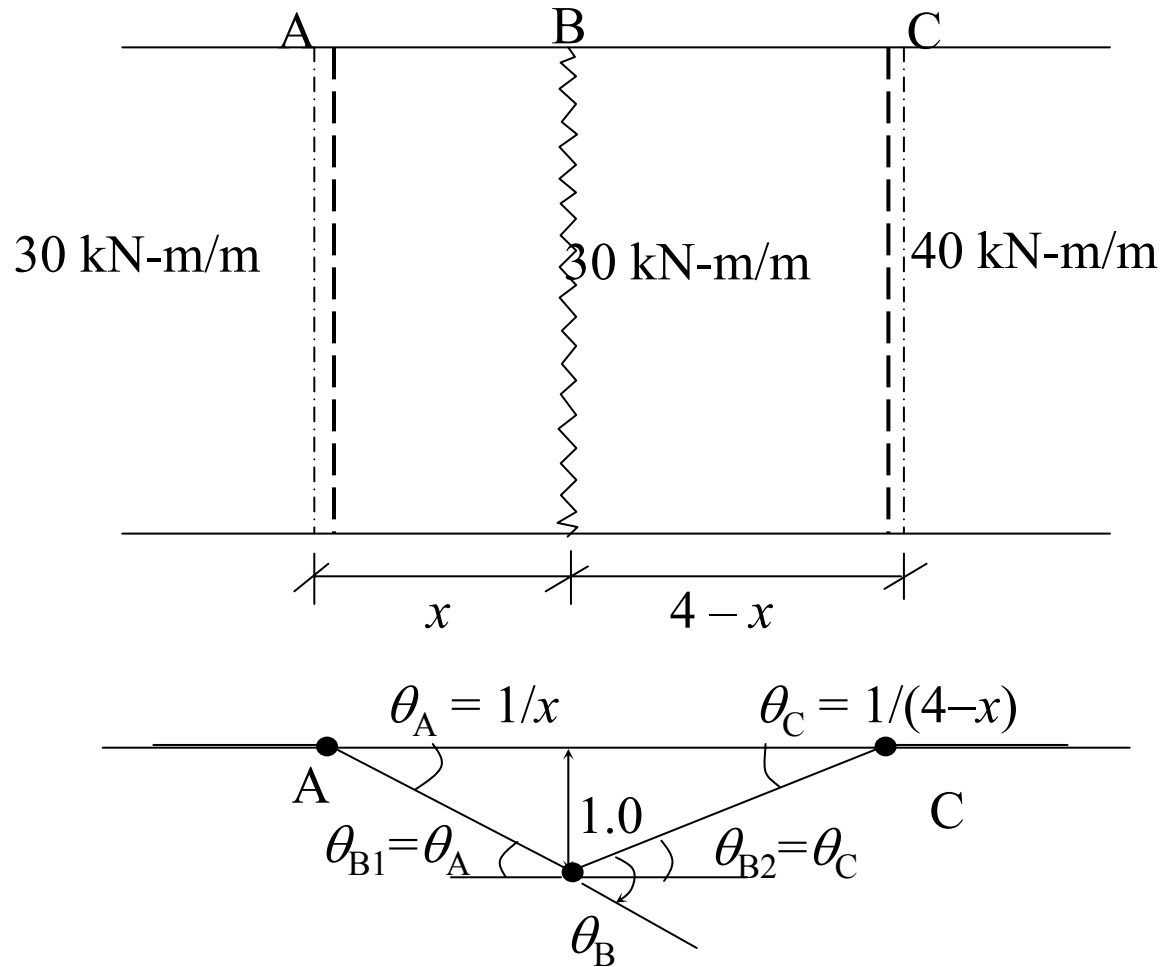


Fig. 13.14. Solution of One-Way Continuous Slab by Virtual Work Method.

Give a unit virtual displacement to yield line-B for the slab system shown in Fig. 13.14.

$$\begin{aligned}\theta_B &= \theta_{B1} + \theta_{B2} \\ &= \frac{1}{x} + \frac{1}{4-x} \\ &= \frac{4-x+x}{x(4-x)} = \frac{4}{x(4-x)}\end{aligned}$$

$$\begin{aligned}W_e &= \frac{1}{2} w x \times 1 + \frac{1}{2} w (4-x) \times 1 \\ &= \frac{w}{2} (x + 4 - x) = 2w\end{aligned}$$



$$\begin{aligned}
 W_i &= (30)\left(\frac{1}{x}\right) + (30)\left(\frac{4}{x(4-x)}\right) + (40)\frac{1}{4-x} \\
 &= \frac{120 - 30x + 120 + 40x}{x(4-x)} = \frac{10x + 240}{x(4-x)}
 \end{aligned}$$

$$W_e = W_i$$

$$2w = \frac{10x + 240}{x(4-x)} \qquad w = \frac{5x + 120}{4x - x^2}$$

To determine the minimum value of the load  $w$ , this expression is differentiated with respect to  $x$  and is equated to zero.

**Formula:** 
$$\frac{d}{dx}(u/v) = \frac{v(du/dx) - u(dv/dx)}{v^2}$$

$$\frac{dw}{dx} = \frac{(4x - x^2)(5) - (5x + 120)(4 - 2x)}{(4x - x^2)^2} = 0$$

$$20x - 5x^2 - 20x + 10x^2 - 480 + 240x = 0$$

$$5x^2 + 240x - 480 = 0$$

$$x^2 + 48x - 96 = 0 \quad \Rightarrow \quad x = 1.923$$

$$\therefore w = \frac{5x + 120}{4x - x^2} = 32.45 \text{ kN/m}^2$$

**Example 13.3:** A square slab is simply supported along all the sides and is to be isotropically reinforced. Determine the resisting moment  $m = \phi m_n$  per linear width required to sustain a uniformly distributed factored load of  $w \text{ kN/m}^2$ . Solve by using both the methods by ignoring the corner lever effects.

## Solution:

### **Segment Equilibrium Method:**

The slab along with the possible yield line pattern without the corner levers is shown in Fig. 13.15.

The collapse load  $w$  is treated as a known, the moment capacity  $m$  is the only one unknown and there is one segment type (giving one equation).

The free body of the slab segment is also shown in the same figure.

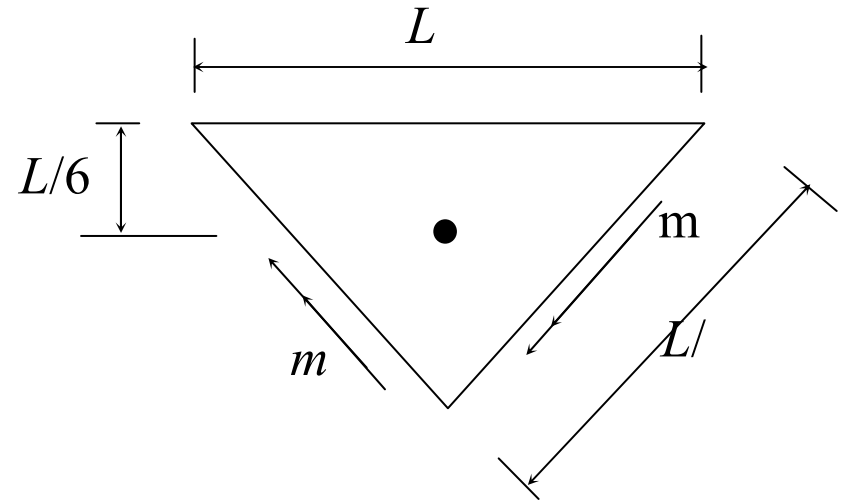
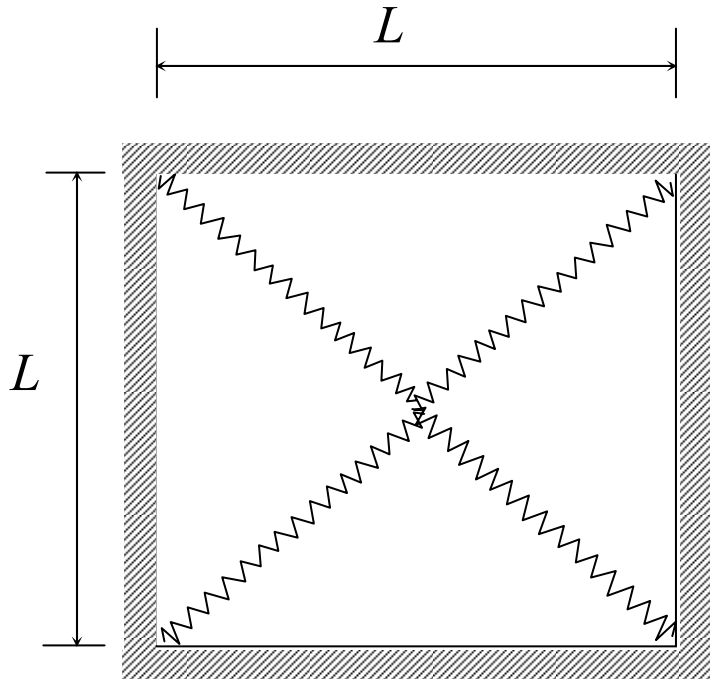


Fig. 13.15. Isotropic Square Simply Supported Slab of Example 13.3.

For sum of moments about the top support line of the free body diagram to be zero:

$$\left(\frac{wL^2}{4}\right) \times \frac{L}{6} = m \frac{L}{\sqrt{2}} \times \left(\frac{1}{\sqrt{2}}\right) \times 2$$

Load on one segment      Total moment on one diagonal edge      To find component of moment along the top edge.      For two edges.

or 
$$\frac{wL^3}{24} = mL$$

The moment about any line may always be considered equal to the projected length of inclined edge along that line multiplied with the moment capacity per unit length along the projected direction.

From above relation:

$$m = \frac{wL^2}{24} \quad \text{per unit width of slab}$$

Alternatively speaking, greater resisting moment is to be provided to carry a given load when corner levers are incorporated in the analysis.

The moment capacity required in this case becomes:

$$m = \frac{wL^2}{21.4}$$

ACI moment value is as under:

$$m = \frac{wL^2}{27.78} \quad (23.0\% \text{ lesser})$$

If deflections are equated in both direction strips, as is done by the British Codes, the value of required moment is as follows:

$$m = \frac{wL^2}{16} \quad (33.7\% \text{ more, safe but uneconomical})$$

The elastic solution obtained by solving the governing differential equation considering homogeneous isotropic material with  $\nu = 0.3$  is as follows:

$$m = \frac{wL^2}{20.9} \quad (2.4\% \text{ more})$$

# Virtual Work Method:

The assumed yield line pattern is shown in Fig. 13.16.

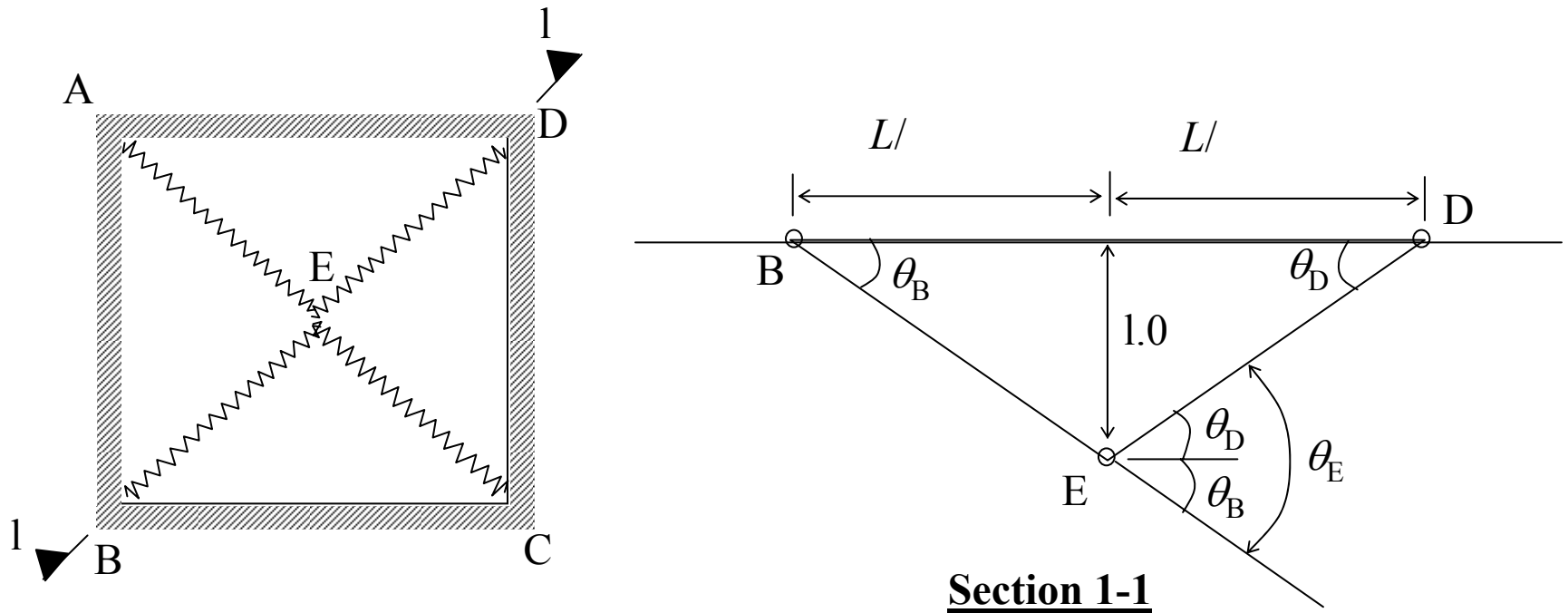


Fig. 13.16. Analysis of Square Simply Supported slab by Virtual Work Method.



$$\begin{aligned}\theta_E &= \theta_B + \theta_D \\ &= \frac{1}{L/\sqrt{2}} + \frac{1}{L/\sqrt{2}} = \frac{2\sqrt{2}}{L}\end{aligned}$$

$$(W_i)_{AE} = m \times L/\sqrt{2} \times \frac{2\sqrt{2}}{L} = 2m$$

$$\text{Total } W_i = 4(2m) = 8m$$

(There are four yield lines similar to AE)

$$\text{Load on plate segment BED} = w \frac{L^2}{4}$$

$$\text{Deflection of centroid of plate BED} = \frac{1}{3}$$

$$W_e \text{ for plate BED} = w \frac{L^2}{4} \times \frac{1}{3} = \frac{wL^2}{12}$$

$$\text{Total } W_e = 4 \times \frac{wL^2}{12} = \frac{wL^2}{3}$$

$$W_e = W_i$$

$$\frac{wL^2}{3} = 8 m \quad m = \frac{wL^2}{24}$$

**Example 13.4:** A two-way slab  $7\text{m} \times 4\text{m}$  is simply supported on all four sides and carries an ultimate UDL of  $w \text{ kN/m}^2$ . Determine the ultimate moment of resistance required for the slab if it is to be isotropically reinforced. Solve by both virtual work and segment equilibrium method by ignoring the corner levers.

## Virtual Work Method:

1. The slab and the expected yield line pattern are shown in Fig. 13.17. Rotation of yield line BE can take place along a line perpendicular to it (GH in this case).
2. The yield line EF is given a unit virtual displacement.
3. The unknowns are  $w$  or  $m$  and  $x$ .
4. Rotation at yield line BE can be found by considering the rotation of two perpendicular lines GE and EH at E.

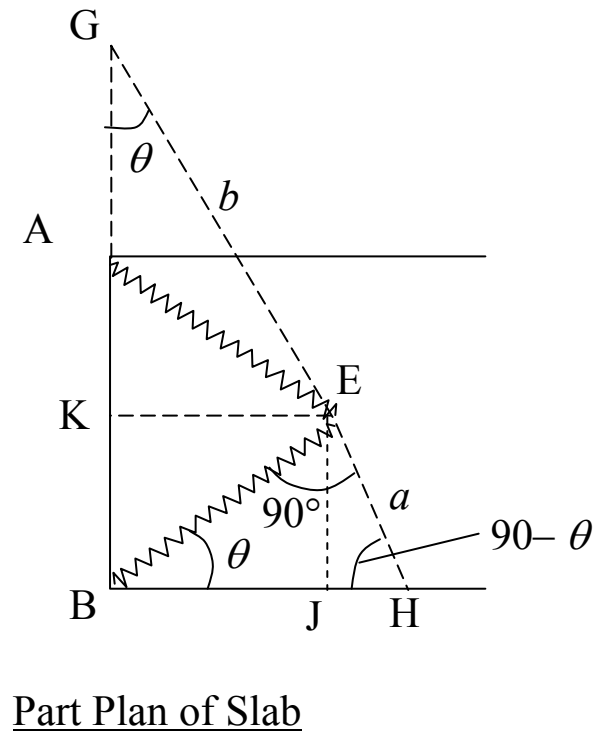
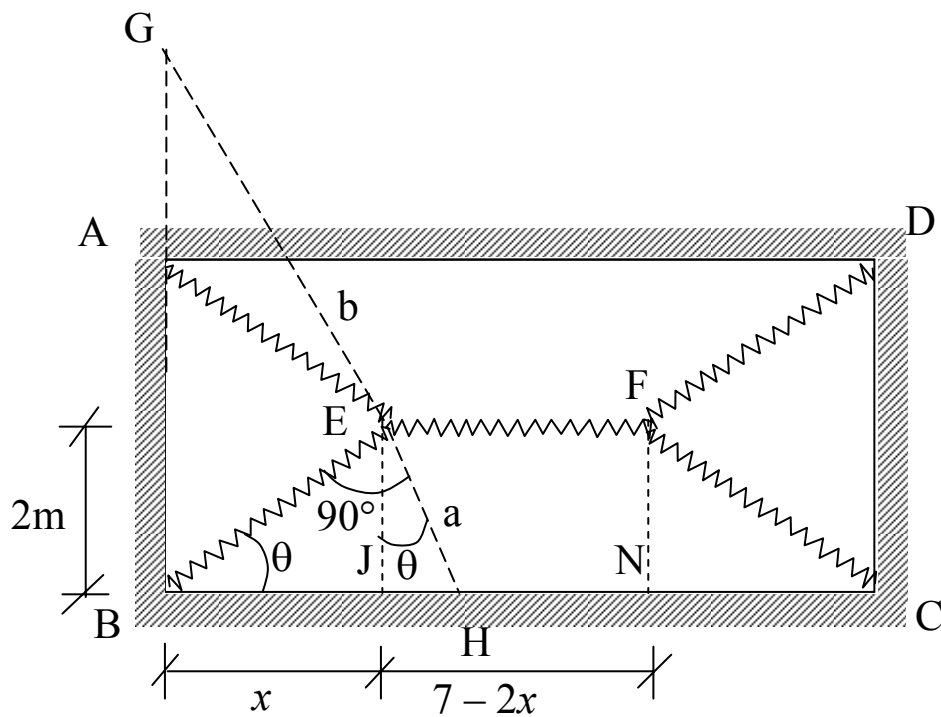


Fig. 13.17. Solution of Slab of Example 13.4 By Virtual Work Method.

$$BE = \sqrt{x^2 + 4}$$

Using similarity of triangles BEJ and GEK:

$$\frac{BE}{EJ} = \frac{GE}{KE} \Rightarrow \frac{\sqrt{x^2 + 4}}{2} = \frac{b}{x}$$

$$\Rightarrow b = \frac{x}{2} \sqrt{x^2 + 4}$$

Triangles BEJ and EHJ are also similar, so,

$$\frac{BE}{x} = \frac{a}{EJ} \Rightarrow a = \frac{\sqrt{x^2 + 4}}{x} \times 2$$

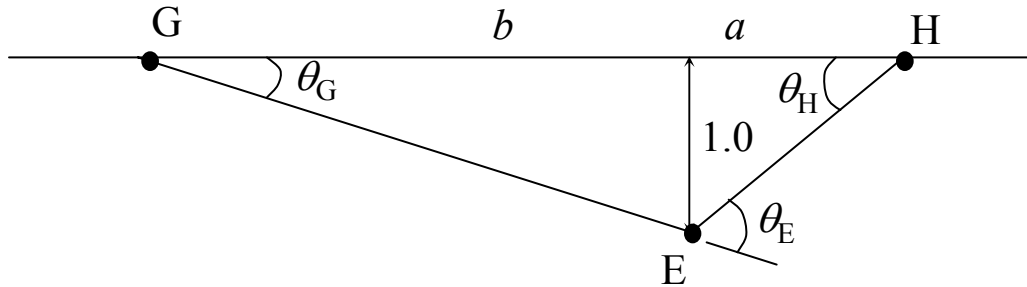


Fig. 13.18. Section GH For Slab of Fig. 13.17.

Considering a section along GH as shown in Fig. 13.18:

$$\begin{aligned}
 \theta_E &= \theta_G + \theta_H \\
 &= \frac{1}{b} + \frac{1}{a} = \frac{2}{x\sqrt{x^2 + 4}} + \frac{x}{2\sqrt{x^2 + 4}} = \frac{\sqrt{x^2 + 4}}{2x}
 \end{aligned}$$

The rotation of yield line EF is:

$$\theta_{EF} = \frac{1}{2} + \frac{1}{2} = 1$$

$$\begin{aligned}
W_i &= W_i \text{ along BE} \times 4 + W_i \text{ along EF} \\
&= \left[ m \times \sqrt{x^2 + 4} \times \frac{\sqrt{x^2 + 4}}{2x} \right] \times 4 + [m(7 - 2x) \times 1] \\
&= \frac{2}{x} m(x^2 + 4) + m(7 - 2x)
\end{aligned}$$

$$\begin{aligned}
W_e &= W_e \text{ due to } \triangle AEB \times 2 + W_e \text{ due to } \triangle BEJ \times 4 \\
&\quad + W_e \text{ due to } \square JEFN \times 2 \\
&= w \frac{1}{2} (4)(x) \left( \frac{1}{3} \right) (2) + w \frac{1}{2} (x)(2) \left( \frac{1}{3} \right) (4) + w(7 - 2x)(2) \left( \frac{1}{2} \right) (2) \\
&= 8/3 w x - 4 w x + 14 w \\
&= -4/3 w x + 14 w
\end{aligned}$$

$$W_i = W_e$$

$$m \left[ \frac{2}{x} (x^2 + 4) + (70 - 2x) \right] = w \left[ -\frac{4}{3}x + 14 \right]$$

$$3m \left[ \frac{2x^2 + 8 + 7x - 2x^2}{x} \right] = w[-4x + 42]$$

$$\frac{w}{m} = \frac{21x + 24}{-4x^2 + 42x}$$

Trial method for minimizing the above value is presented in Table 13.1.



Table 13.1. Trial Method to Calculate Collapse Load.

$x$	$\frac{w}{m}$
2.0	0.97059
2.2	0.96112
2.4	0.95679
2.5	0.95625
2.55	0.95634
2.52	0.95626

Hence,  $x \approx 2.51 \text{ m}$

and  $\frac{w}{m} \approx 0.95625 \quad \therefore m = 1.046 w$

### **Segment Equilibrium Method:**

There are two unknowns and two distinct segments in this example.

These slab segments are shown in Fig. 13.19.

Considering equilibrium of segment AEB, we get,

$$12 m = \frac{4x}{2} w \times \frac{x}{3} \quad m = \frac{wx^2}{6} \quad (\text{I})$$

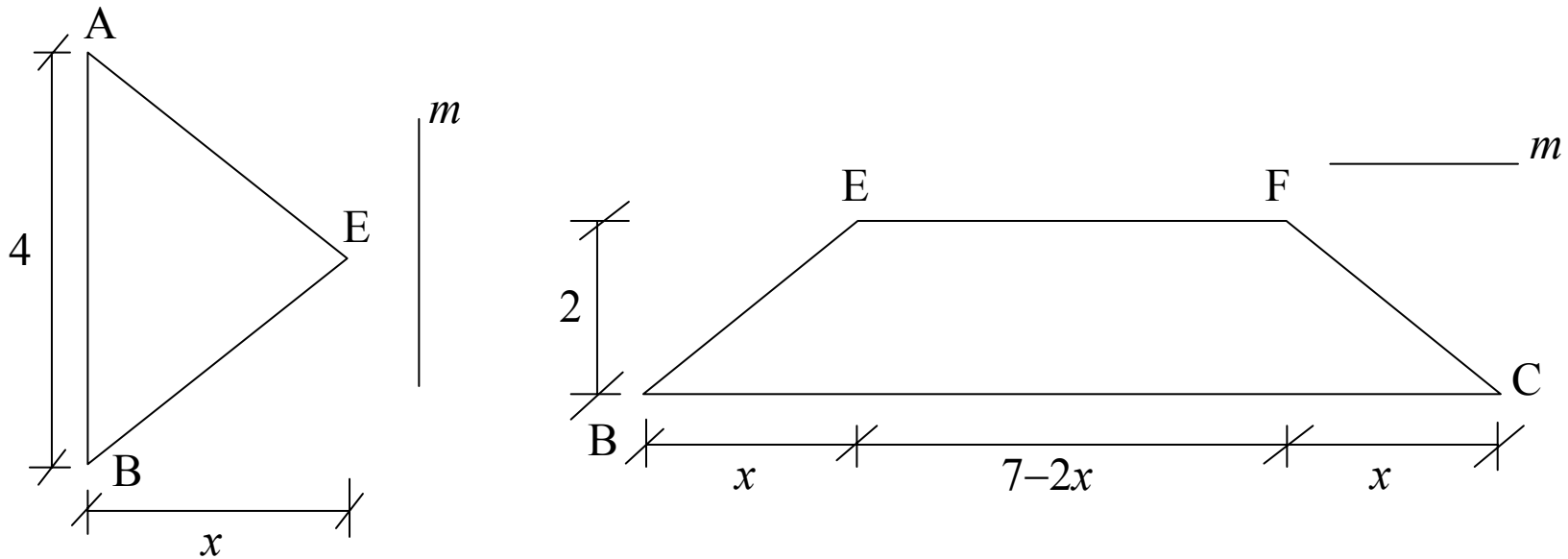


Fig. 13.19. Free Body Diagrams For Slab of Example 13.4.

Considering equilibrium of segment BEFC,

$$7 m = 2 \times \left(\frac{1}{2}\right) \times 2 \times x \times w \left(\frac{2}{3}\right) + (2)(7 - 2x) w \left(\frac{2}{2}\right)$$

$$7 m = -8/3 w x + 14 w \quad \text{(II)}$$

Using the value of  $m$  from Eq. II in Eq. I, we get:

$$7x^2 + 16x - 84 = 0, \quad x = 2.505 \text{ m}$$

$$\text{From Eq. (I),} \quad m = 1.046 w$$

## **SPECIAL CONDITIONS AT EDGES AND CORNERS**

Positive yield lines in some of the patterns shown earlier, where these have to intersect free edges not at right angles as shown in Fig. 13.25b, are assumed to go straight and intersect the edges at angles different from the right angle.

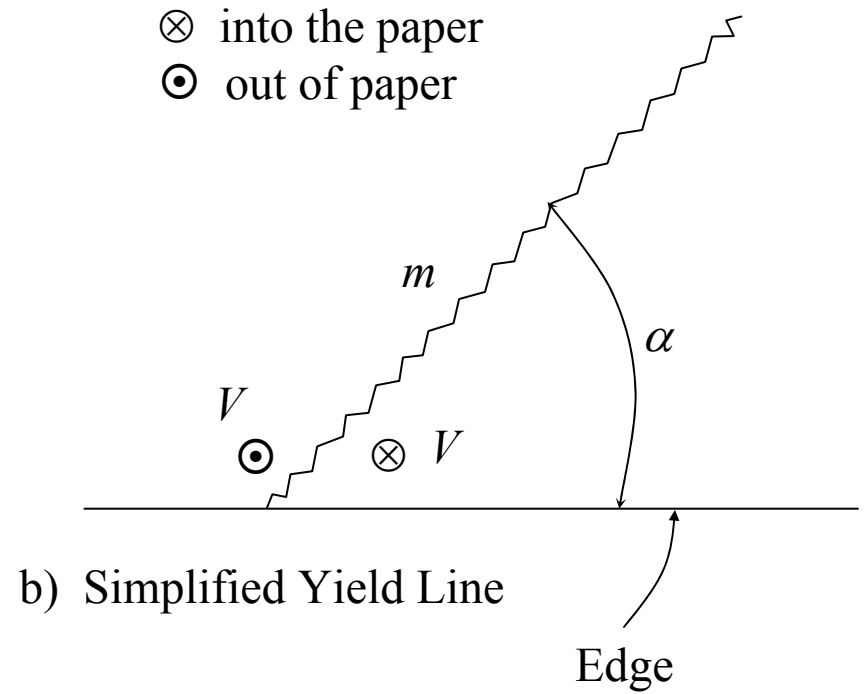
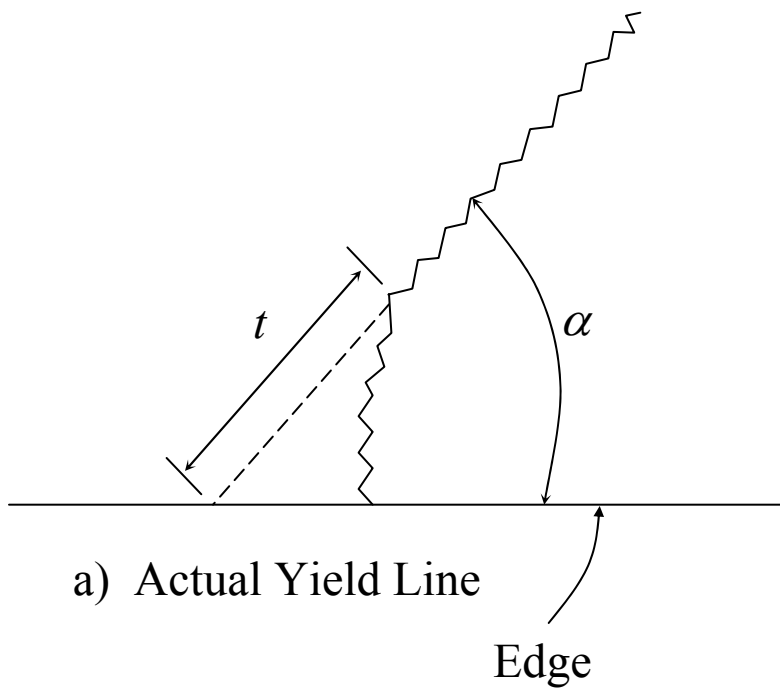


Fig. 13.25. Positive Yield Line at a Free Edge.

This is just a simplification at an edge.

At a free edge, both the bending moment and the twisting moment should be zero.

The major principal direction is perpendicular to the edge, the bending moment having moment vector in this direction should be the maximum and yield line must be formed in this direction.

Consequently, the yield lines should enter an edge perpendicular to it, as shown in Fig. 13.25a.

This is a contradiction with the original statement that the yield lines must be straight, and is to be treated as a special case.

Tests confirm that yield lines change their angle very close to the edge and enter the edge in a perpendicular position.

The distance ' $x$ ' in the figure is usually much small compared to the other slab dimensions.

The calculation procedure may be simplified by still considering straight yield lines, as shown in Fig. 13.25a, which increases the collapse load due to the use of wrong assumption.

This collapse load may artificially be may made smaller equal to the actual collapse load by introduction of an applied twisting moment at the edge.

This twisting moment not only overload the slab but also creates the condition of maximum bending moment along the inclined direction of the yield line.

Further, in place of applying this twisting moment, a shear force  $V$  acting downward at the acute corner (*circled cross* in the figure) and the force  $V$  acting upward at the obtuse corner (*circled dot* in the figure) together are applied as the static equivalent of twisting moment.

The magnitude of this fictitious shearing forces  $V$  is given by the following expression:

$$V = m \cot \alpha$$



Where ' $m$ ' is the resisting moment per unit length along the yield line and ' $\alpha$ ' is the acute angle between simplified yield line and the edge of the slab.

This pair of equal and opposite forces does not enter the virtual work calculations as the work done by both of them cancels each other.

However, when free body diagram of each segment is separately considered in the equilibrium method, the forces  $V$  enter in to the calculations.

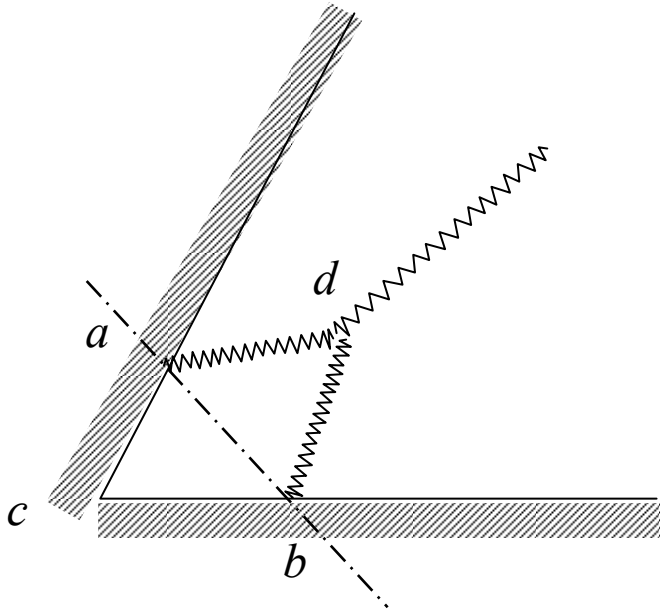
To simplify the calculations at corners, it is commonly assumed that yield line enters the corner as a single straight line passing between the two intersecting sides.

In reality, the yield lines fork or divide into two branches to form a Y before reaching the corners.

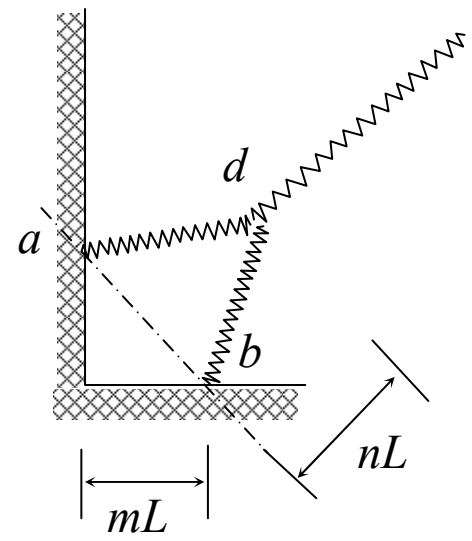
The resulting slab segment formed near the corner is called a *corner lever* or *corner pivot*, as shown in Fig. 13.26.

If the corner is not held down, the triangular element  $adb$  will pivot about the axis  $ab$  and lift off the supports.

If the corner is held down, the line  $ab$  becomes a negative yield line and the segment  $abd$  rotates about it.



a) Corner Allowed to Lift-Up



b) Corner Held Down

Fig. 13.26. Formation of Corner Levers.

Consider the free body of the corner lever formed shown in Fig. 13.27.

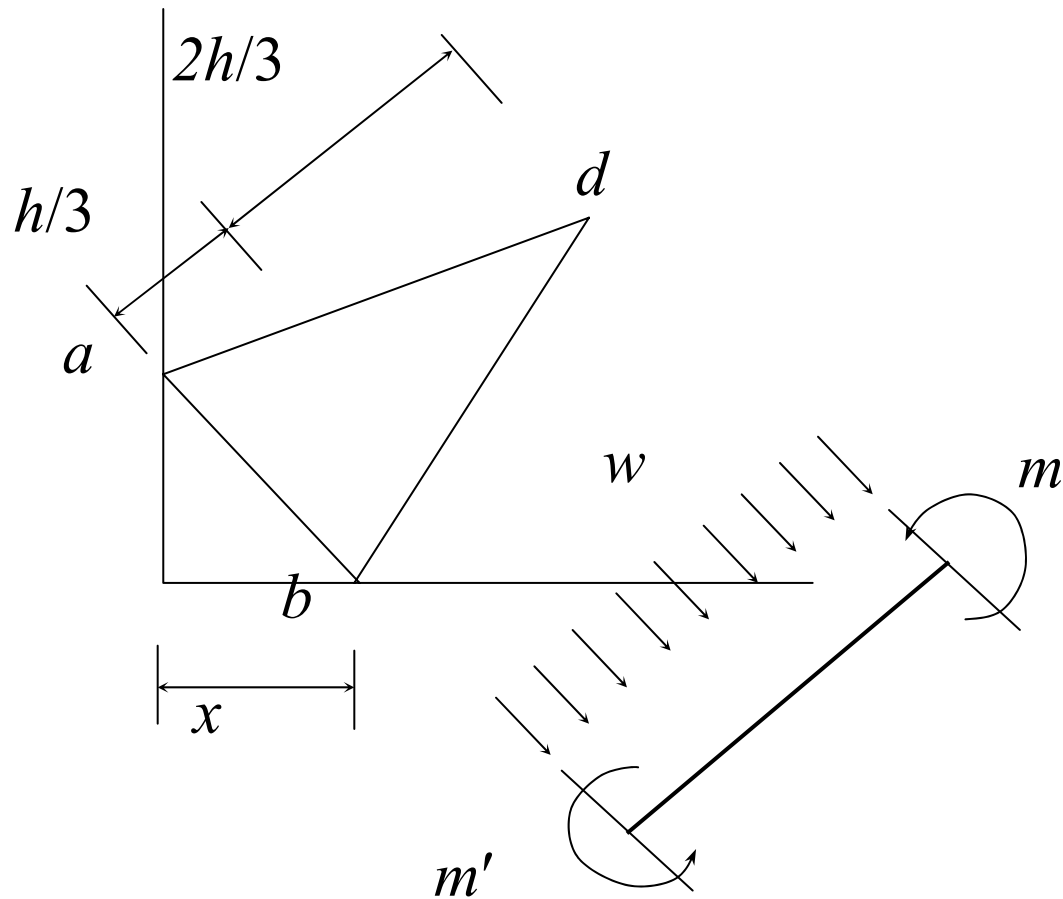


Fig. 13.27. Free Body Diagram of Corner Lever.

If no shear force acts at the apex of the segment, we get,

$$\begin{aligned}\Sigma M_{ab} = & -1.414 x m' - 1.414 x m \\ & + 0.5 (1.414 x)(w)(h)(h/3) = 0\end{aligned}$$

$$\frac{wh^2}{6} = m + m' \quad \therefore h = \sqrt{6(m + m') / w}$$

Hence, the distance of junction of the Y from the pivot,  $h$ , is independent of the distance  $x$  and the width of the segment.

It increases as both the positive and negative moment capacities increase with respect to the load intensity.

Although yield line patterns with corner levers are generally more critical than those without them, they are often neglected and only approximate adjustment is made for their effect.

In square panels, the maximum effect of the corner segments is slightly less than 9%.

Further, if sufficient corner steel is separately provided, these can be eliminated in the strength calculations.

**Example 13.8:** For the isotropically reinforced slab shown in Fig. 13.28, find the collapse load in terms of the moment using virtual work method.

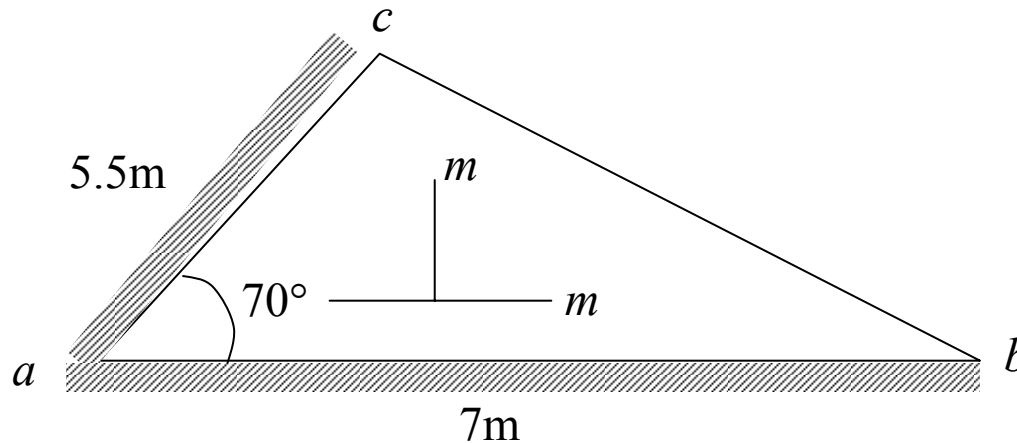
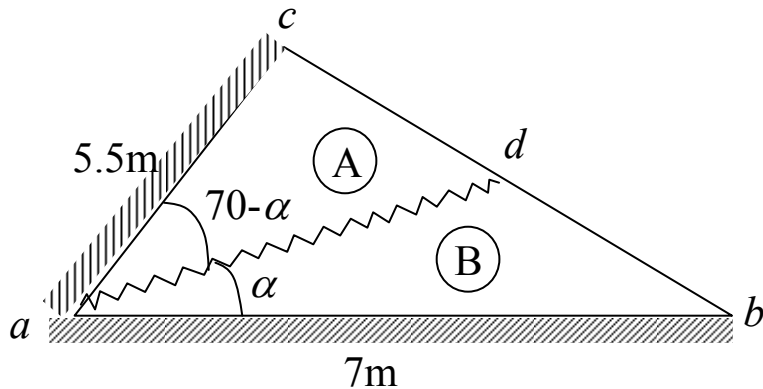


Fig. 13.28. Triangular Slab For Example 13.8.

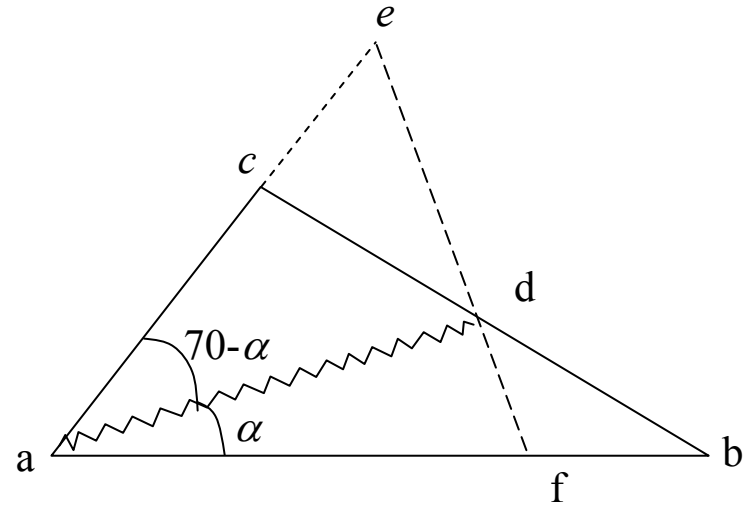
### **Solution:**

The selected yield-line pattern is shown in Fig. 13.29, which involves two unknowns in the form of collapse load and the angle  $\alpha$ .

A unit vertical deflection is applied at the point ' $d$ '.



a) Possible Yield-Line Pattern



b) Geometric Properties of the Yield-Line Pattern

Fig. 13.29. Yield Line Pattern For Slab of Example 13.8.

$$\begin{aligned}
 W_e &= w \times \text{area of } \Delta A \times 1/3 + w \times \text{area of } \Delta B \times 1/3 \\
 &= w/3 \times \text{total area} \\
 &= w/3 (\text{base})(\text{height})^{1/2} \\
 &= [w/6] (7)(5.5 \sin 70^\circ) \\
 &= \frac{77}{12} w \sin 70^\circ
 \end{aligned}$$



Consider a section perpendicular to the Y.L, shown as  $ef$  in Fig. 13.29(b).

$$ed = ad \tan (70 - \alpha)$$

$$df = ad \tan (\alpha)$$

$$\theta_e = 1/ed$$

$$\theta_f = 1/df$$

$$\theta_d = \text{rotation all along the yield line}$$

$$= \theta_e + \theta_f$$

$$= \frac{1}{ad \tan(70 - \alpha)} + \frac{1}{ad \tan \alpha}$$

$$= \frac{1}{ad} (\cot(70 - \alpha) + \cot \alpha)$$

$W_i$  = moment capacity at Y.L.  $\times$  length of  
Y.L.  $\times$  rotation at Y.L.

$$= m \times ad \times \frac{1}{ad} [\cot(70 - \alpha) + \cot \alpha]$$

$$= m [\cot(70 - \alpha) + \cot \alpha]$$

$$W_e = W_i$$

$$\Rightarrow \frac{77}{12} w \sin 70^\circ = m [\cot(70 - \alpha) + \cot \alpha]$$

Trigonometric Relation:  $\cot A + \cot B = \frac{\sin(A + B)}{\sin A \sin B}$

$$\therefore \quad {}^{77/}_{12} w \sin 70^\circ = m \frac{\sin (70 - \alpha + \alpha)}{\sin (70 - \alpha) \sin \alpha}$$

$$\text{or} \quad \frac{m}{w} = {}^{77/}_{12} \sin (70 - \alpha) (\sin \alpha) \quad (\text{I})$$

To maximize  $\frac{m}{w}$ , we have the relation  $\frac{d\left(\frac{m}{w}\right)}{d\alpha} = 0$

$${}^{77/}_{12} [-\cos (70 - \alpha) \sin \alpha + \sin (70 - \alpha) \cos \alpha] = 0$$

$$\sin \alpha \cos (70 - \alpha) = \sin (70 - \alpha) \cos \alpha$$

$$\tan \alpha = \tan (70 - \alpha) \quad \alpha = 70 - \alpha$$

$$\Rightarrow 2\alpha = 70^\circ \Rightarrow \alpha = 35^\circ$$

From Eq. I:  $\frac{m}{w} = \frac{77}{12} \sin^2 35^\circ$

$w = 0.4737 m$

**Example 13.9:** For the orthotropically reinforced slab shown in Fig. 13.30, find the collapse load using segment equilibrium method.

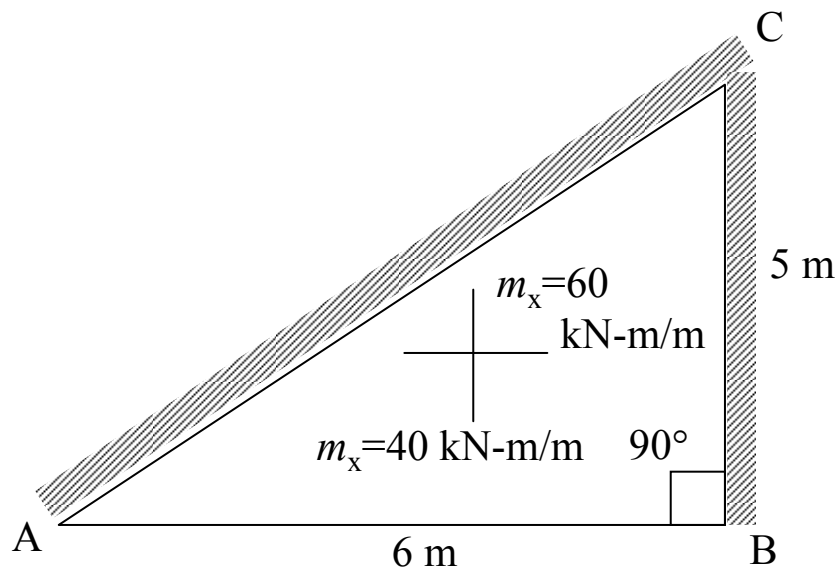


Fig. 13.30. Triangular Slab For Example 13.9.

**Solution:** The selected yield-line pattern is shown in Fig. 13.31 along with the free body diagrams of the two segments formed.

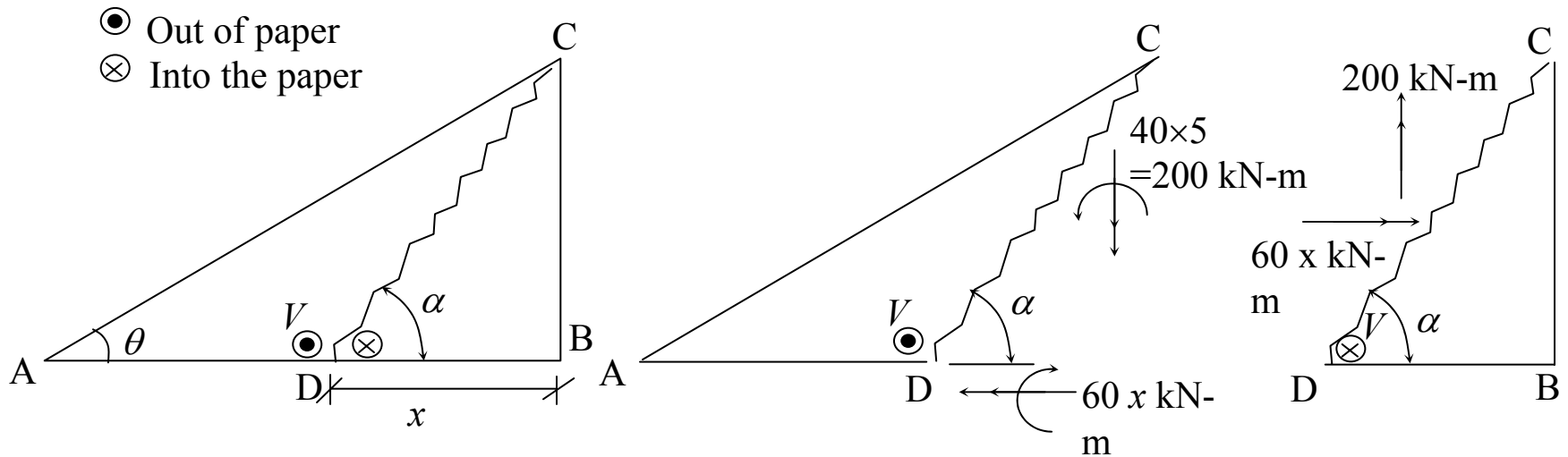


Fig. 13.31. Yield Line Pattern For Slab of Example 13.9.

Let  $x$  = distance of point of intersection of yield line with free edge measured from point B.  
 and  $\alpha$  = acute angle of yield line with respect to the free edge.

$$\cos \alpha = \frac{x}{\sqrt{x^2 + 25^2}} \quad \text{and} \quad \sin \alpha = \frac{5}{\sqrt{x^2 + 25^2}}$$

$$\cot \alpha = \frac{x}{5} \quad \theta = \tan^{-1} \left( \frac{5}{6} \right) = 39.8^\circ$$

Along the yield line CD, the following may be calculated:

$$\begin{aligned} m &= m_n = m_x \cos^2 \alpha + m_y \sin^2 \alpha \\ &= 60 \frac{x^2}{x^2 + 25^2} + 40 \frac{25}{x^2 + 25} = \frac{60x^2 + 1000}{x^2 + 25} \end{aligned}$$

$$\begin{aligned} V &= m \cot \alpha \\ &= m \times \frac{x}{5} = \frac{12x^3 + 200x}{x^2 + 25} \end{aligned}$$

## Segment ACD

It is to be noted that the horizontal and vertical moments at the yield line may directly be resolved along the line about which moment sum is to be taken, as shown in Fig. 13.32.

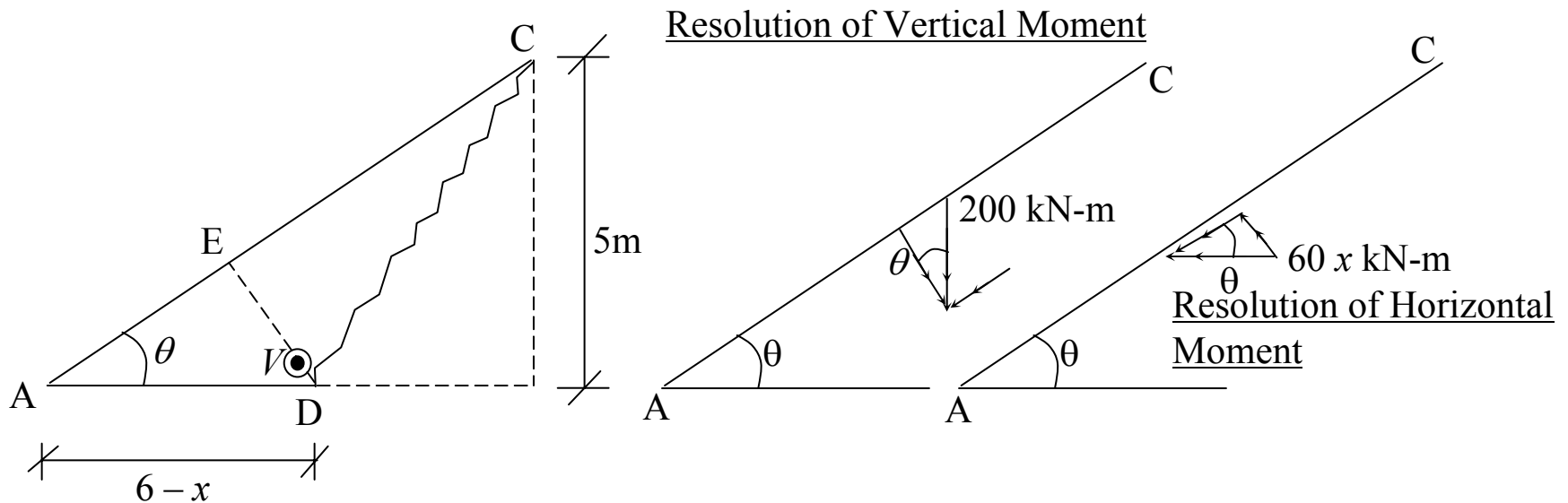


Fig. 13.32. Resolution of Horizontal and Vertical Moments At Y.L. About Line AC.

Referring to Fig. 13.32, we have,

$$DE = (6 - x) \sin\theta$$

$$\sin\theta = BC / AC = 5 / AC \quad : \quad \tan\theta = 5/6$$

$$\Sigma M_{AC} = 0 \Rightarrow$$

$$w \left[ \frac{1}{2} (DE)(AC) \right] \times \frac{DE}{3} - V \times DE = \text{components of moments along yield line CD about AC}$$

$$\begin{aligned} \frac{w}{6} \times AC \times (6 - x)^2 \times \frac{5}{AC} \times \sin\theta - \frac{12x^3 + 200x}{x^2 + 25} (6 - x)(\sin\theta) \\ = 200 \sin\theta + 60 x \cos\theta \end{aligned}$$



$$\frac{5w}{6}(6-x)^2 - \frac{12x^3 + 200x}{x^2 + 25}(6-x) = 200 + 60x / \tan\theta$$

$$\frac{5w}{6}(6-x)^2 = (200 + 72x) - \frac{20x^3 + 200x}{x^2 + 25}(6-x)$$

$$w = \frac{240 + 86.4x}{(6-x)^2} + \frac{14.4x^3 + 240x}{(x^2 + 25)(6-x)} \quad (\text{I})$$

## **Segment DBC**

The bending moment at the yield line is already resolved into horizontal and vertical components and moment is to be taken about the vertical line. Hence, there is no need to further resolve the moments.

$$\Sigma M_{BC} = 0 \quad \Rightarrow$$

$$w \left( \frac{1}{2} \times x \times 5 \right) \times \frac{x}{3} + V \times x = 200$$

$$w \frac{5x^2}{6} + \frac{(12x^4 + 200x^2)}{(x^2 + 25)} = 200$$

$$w = \frac{240}{x^2} - \frac{(14.4x^2 + 240)}{(x^2 + 25)} \quad (\text{II})$$

By solving Eqs. I and II simultaneously using trial method, as in Table 13.2, we get,

$$w = 37.07 \text{ kN/m}^2$$

Table 13.2. Solution of Example 13.9.

$x$ (m)	$w$ from Eq. I (kN/m <sup>2</sup> )	$w$ from Eq. II (kN/m <sup>2</sup> )
3.0	66.34	15.80
2.5	44.77	27.84
2.0	30.93	49.74
2.2	35.79	39.21
2.25	37.14	37.00



$$\begin{aligned} \text{Total load} &= P \\ \text{Load per unit angle in radians} &= \frac{P}{2\pi} \\ \text{Load per angle equal to } \beta &= \frac{P}{2\pi} \beta \end{aligned}$$

In Fig. 13.33, the projected length of positive yield lines  $AC$  and  $CB$  is the length  $AB$  (or  $r\beta$ ) and hence the total component of positive moment about  $XX$ -axis is  $m r\beta$ .

The total negative moment about axis- $XX$  is  $m^-r\beta$ .

Both the negative and the positive moments are counter-clockwise.

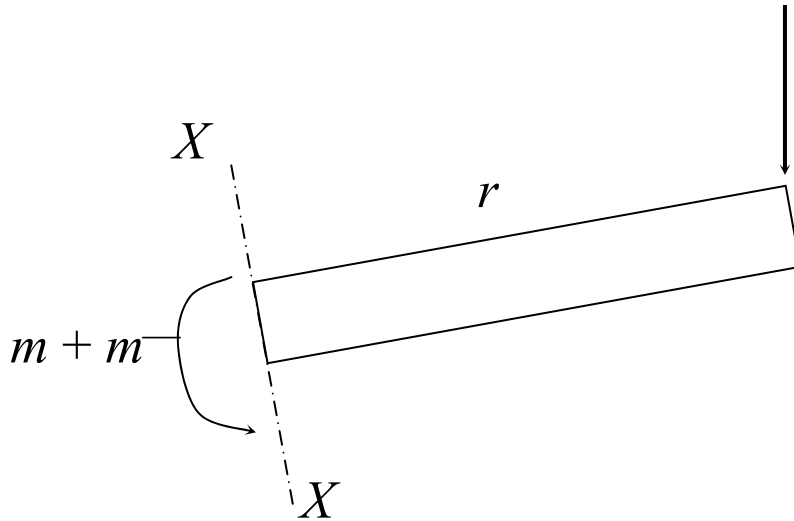


Fig. 13.34. Sectional Front View of a Radial Strip Through Fan Pattern.

Referring to Figs. 13.33 and 13.34, we have,

$$\Sigma M_{XX} = 0 \quad \Rightarrow$$

$$m^- r \beta + m r \beta = \frac{P \beta}{2\pi} r$$

$$P = 2 \pi (m + m^-)$$

The collapse load  $P$  is seen to be independent of the fan radius  $r$ .

With only concentrated load acting, a complete fan of any radius could form with no change in collapse load.

A fan pattern will be controlling failure mechanism if the collapse load for this mechanism is the minimum of collapse loads for all other possible yield lines.

## **AFFINITY RULES**

*Affine slab* is an idealized slab considered for analysis in place of an orthotropic (different moment capacities in two perpendicular directions) and/or skew (sides of the slab or reinforcement not at right angles) slab.

The affine slab is obtained by transforming the shape and loading of the orthotropic slab according to certain rules, given below:

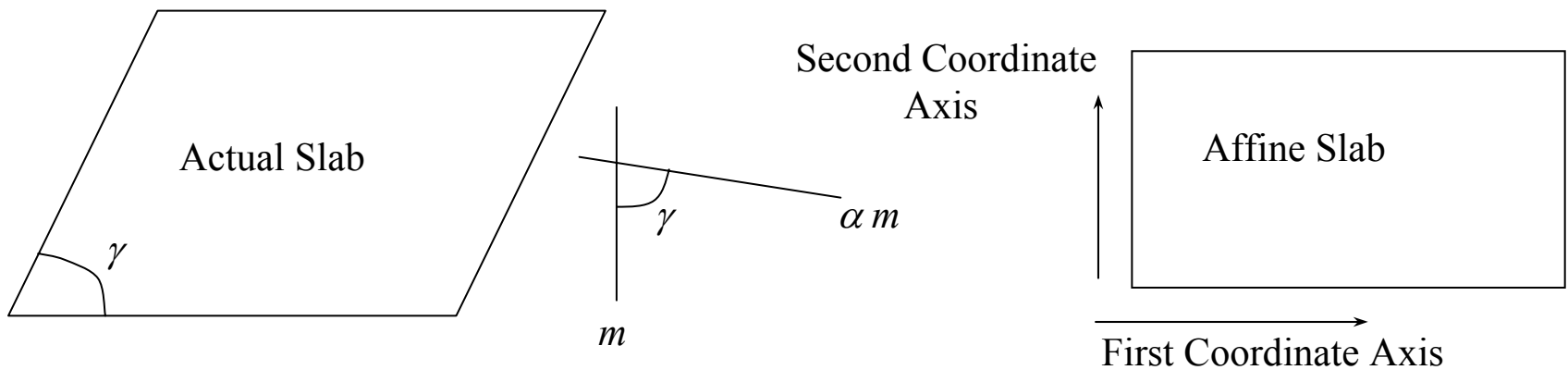


Fig. 13.35. Orthotropic And Corresponding Affine Slab.



1. The original slab must have reinforcement in two directions parallel to the edges of the slab (separated by an angle  $\gamma$ ), such that the ratio of the ultimate moments due to each set of reinforcement taken separately is constant throughout the slab.
2. The deflections at the corresponding points of the skew and affine slabs are considered equal.
3. The strength of the equivalent isotropic slab at a point corresponding to the actual slab is considered equal to  $m$  in all directions.

4. The affine slab is drawn such that all distances measured in the direction of the  $m$ -reinforcement (perpendicular to the  $m$  moment vector) remain the same. This direction forms one co-ordinate axis for both slabs called the *first coordinate axis*.
5. The second direction in the skew slab, corresponding to the  $\alpha m$ -reinforcement direction, is taken at right angles to the first coordinate axis in the affine slab, and is called the *second coordinate direction*.

6. All distances in the affine slab in this coordinate direction are obtained by dividing corresponding lengths in the skew slab by  $\sqrt{\alpha}$ .
7. All loads in the affine slab are obtained by modifying the original loads as under: -
  - a. Uniformly distributed load  $w_u$  remains  $w_u$  in the affine slab.
  - b. Point load  $P_u$  is replaced by  $\frac{P_u}{\sqrt{\alpha} \times \text{Sin} \gamma}$  in the affine slab.