## **Direct Two-Way Or Punching Shear Force:**

The direct shear force, $V_u$, to be resisted by the slab-column connection can be calculated as the total factored load on the area bounded by panel centerlines around the column less the load applied within the area defined by the critical shear perimeter.

This is to be calculated both at the column perimeter and at the perimeter of drop panel, if present, using the critical section defined in Figs. 12.18 and 12.19.

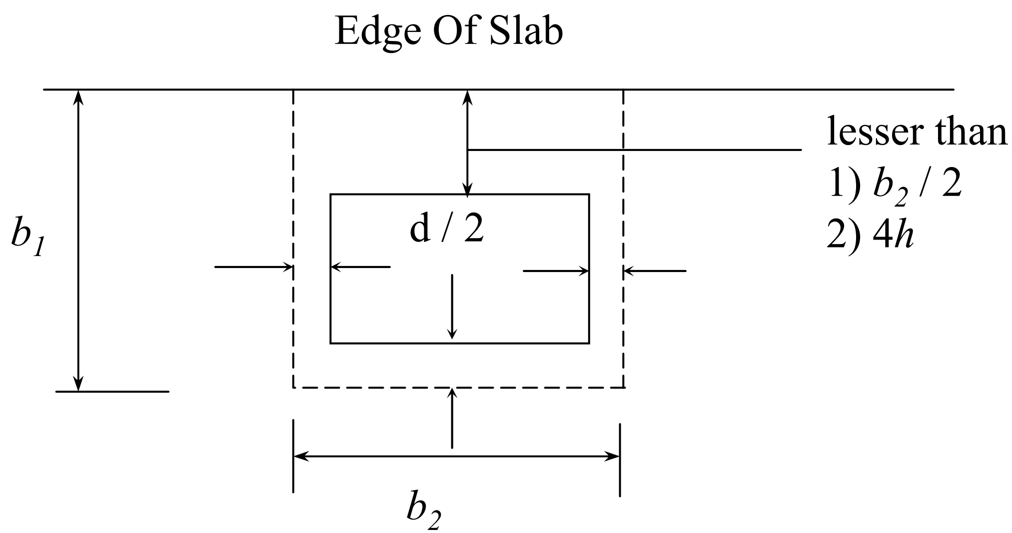
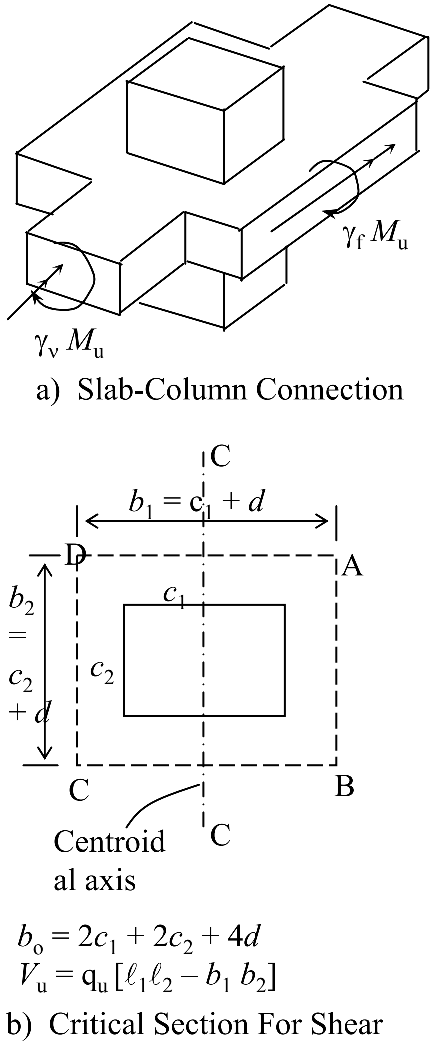



Fig. 12.19. Critical Section For Edge Column.

## **Eccentric Punching Shear Force:**

According to ACI 11.2.6.2, the shear stress resulting from moment transfer by eccentricity of shear shall be assumed to vary linearly about the centroid of the critical section.



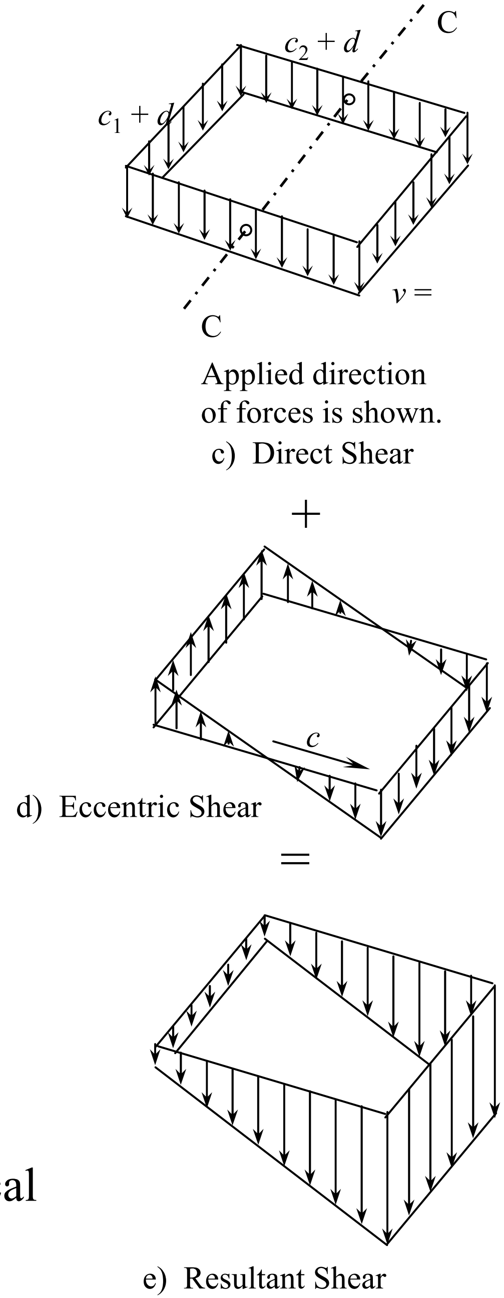




Fig. 12.20. Two-Way Shear Acting on Critical Slab Section Around Column.

The resultant shear stress acting on the critical perimeter, considering moment acting from both the directions, may be written as follows:

$$v_{\mathrm{u}} \text{ at face AB } = \frac{V_u}{A_c} \pm \frac{(\gamma_u M_u)_1 \left(\frac{b_1}{2}\right)}{J_{c_1}} \pm \frac{(\gamma_u M_u)_2 \left(\frac{b_2}{2}\right)}{J_{c_2}}$$

Where, $A_{\mathrm{c}}$ and $J_{\mathrm{c}}$ are calculated for the faces of a box-like shape defined by the assumed vertical failure section.

$A_{\mathrm{c}}$ = perimeter area of the critical section

= $b_{\mathrm{o}}d$ = $2(b_1 + b_2)\, d$

$J_c$ = torsional constant, like polar moment of inertia of the area $A_c$

$= I_x + I_y$

**<u>Torsional Constant For Interior Column:</u>**

The critical area subjected to punching shear is a three dimensional area and hence the calculation of its torsional constant is not as simple as for any planar area.

In order to get a reasonably good estimate, the width of the area may be squeezed to zero but maintaining the original area.

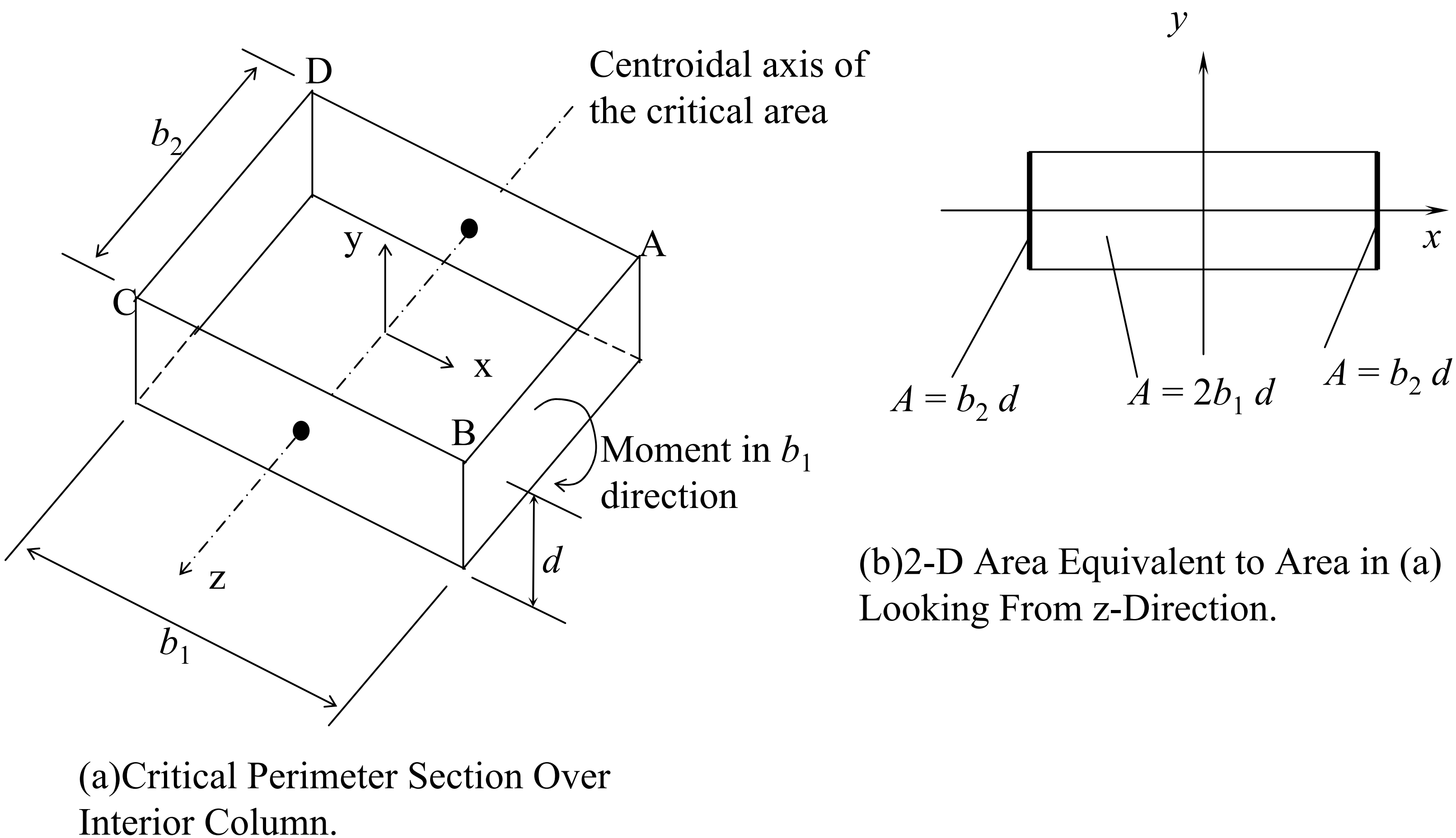



Fig. 12.21. Critical Section For Two-Way Shear Over Interior Column.

$J_c = I_x$ for faces AD and BC $+\ I_x$ for faces AB and CD $+\ I_y$ for faces AD and BC $+\ I_y$ for faces AB and CD

$$= 2 \times \left[\frac{b_1 d^3}{12} + 0\right] + 0 + 2 \times \left[\frac{d b_1^3}{12} + 0\right] + 2 \times \left[(b_2 d)\left(\frac{b_1}{2}\right)^2\right]$$

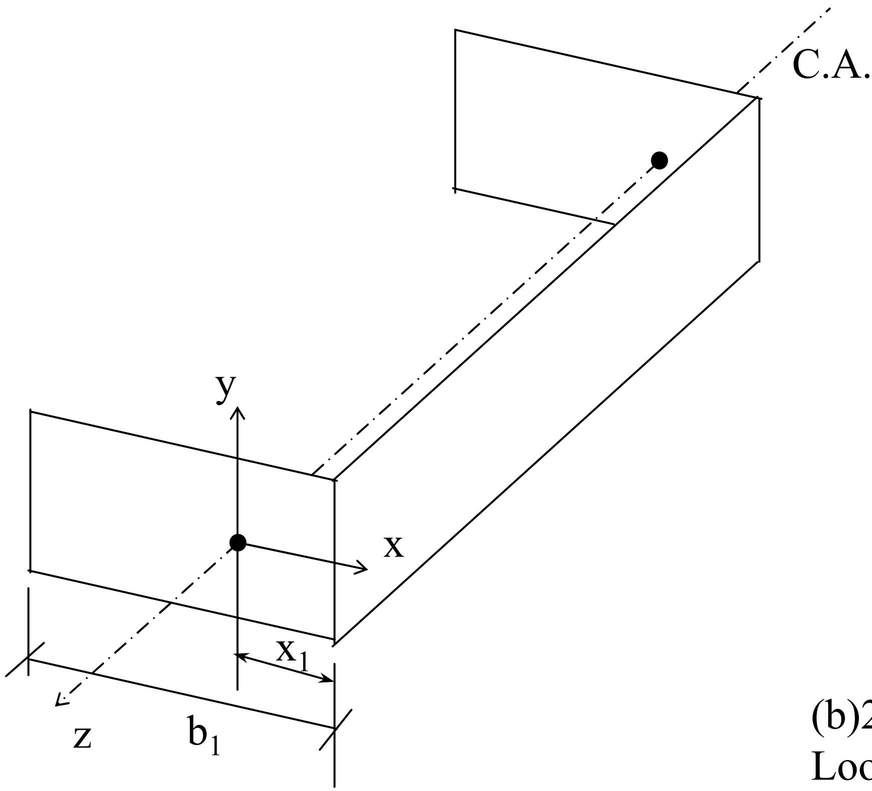
$$= \frac{b_1 d^3}{6} + \frac{d b_1^3}{6} + \frac{d b_2 b_1^2}{2}$$

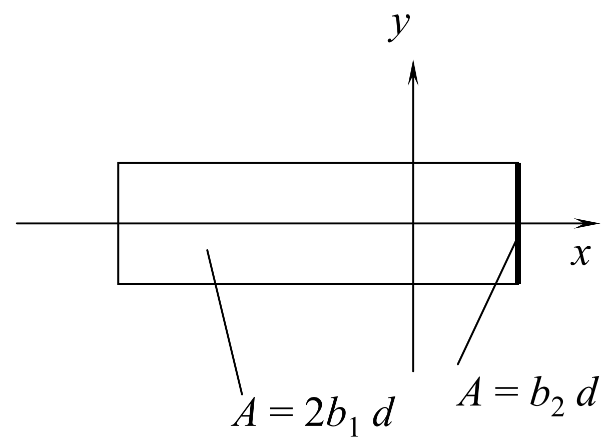
## Torsional Constant For Edge Column:

$$x_1 = \frac{2 b_1 d \left(b_1 / 2\right)}{2 b_1 d + b_2 d} = \frac{b_1^2}{2 b_1 + b_2}$$

$$A_c = (2 b_1 + b_2)\, d$$

$$J_c = 2\times\left\{\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + b_1 d\left(\frac{b_1}{2} - x_1\right)^2\right\} + (b_2 d)(x_1)^2$$

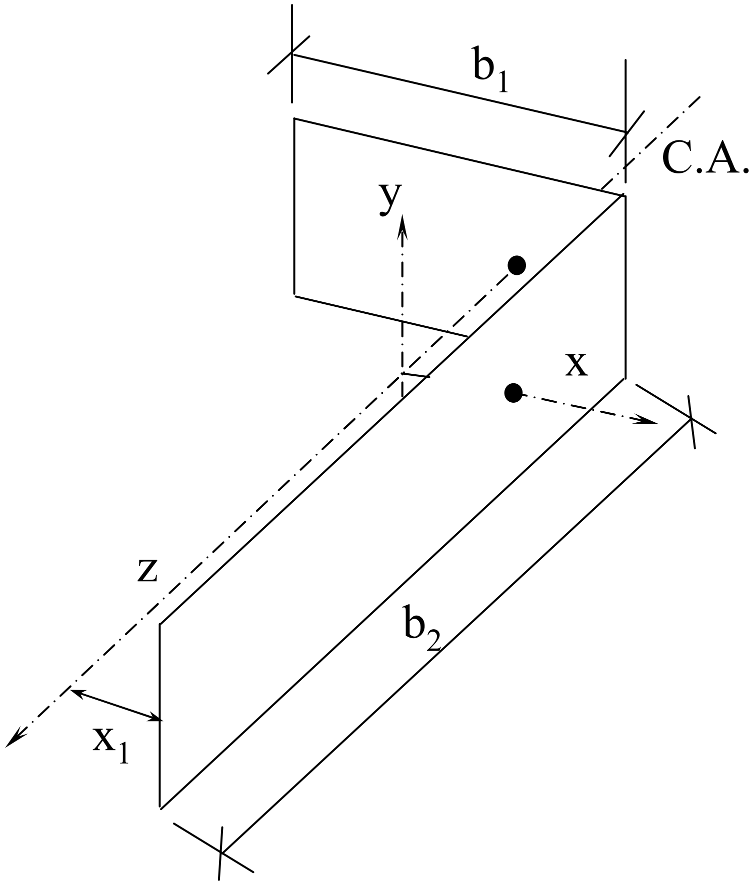


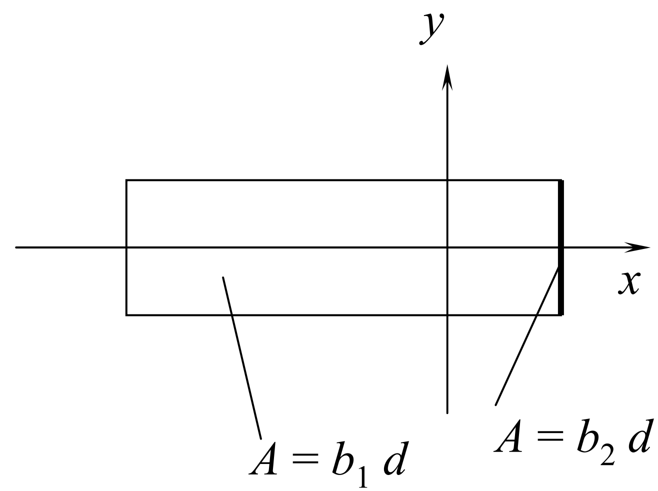




(b)2-D Area Equivalent to Area in (a)
Looking From z-Direction.

Fig. 12.22. Critical Section For Two-Way Shear Over Edge Column.

# Torsional Constant For Corner Column:




(b)2-D Area Equivalent to Area in (a)
Looking From z-Direction.

Fig. 12.23. Critical Section For Two-Way Shear Over Corner Column.

$$x_1 = \frac{b_1 d \left(b_1/2\right)}{b_1 d + b_2 d} = \frac{b_1^2}{2b_1 + b_2}$$

$$A_c = (b_1 + b_2)\, d$$

$$J_c = \frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + b_1 d \left(\frac{b_1}{2} - x_1\right)^2 + \left(b_2 d\right)\left(x_1\right)^2$$

## Concrete Punching Shear Strength ($V_c$):

According to ACI 11.12.2.1, for non-prestressed slabs and footings, $V_c$ shall be the smallest of:

a) $$V_c = \frac{1}{6}\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'}\; b_o d$$

$$\beta = \frac{\text{long side}}{\text{short side}}$$ of column, concentrated load, or section area.

$b_o$ = perimeter of critical section for slabs and footings.

b) $V_c = \frac{1}{12}\left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c}\; b_o d$

where $\alpha$ = *40* for interior columns

= *30* for edge columns

= *20* for corner columns

c) $V_c = \frac{1}{3}\sqrt{f'_c}\; b_o d$

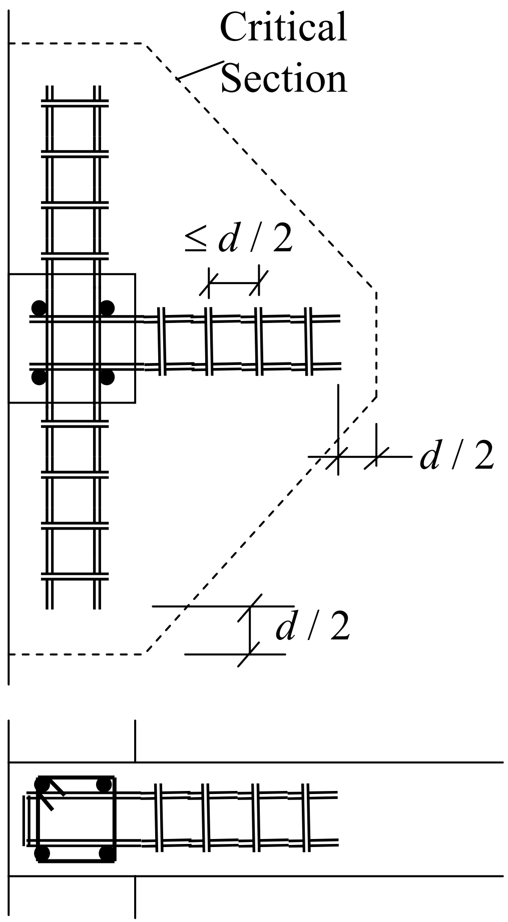
For design, maximum shear stress due to the factored shear force and moment $\leq \phi v_n$

where $\phi v_n = \phi(V_c + V_s)/(b_o d)$

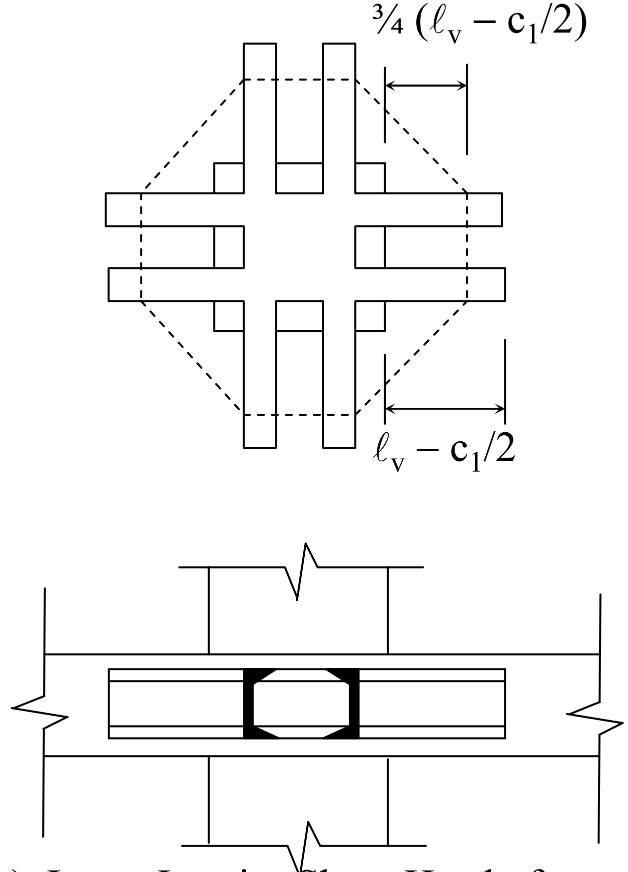
$V_s = 0$, when no shear reinforcement in slabs in the form of shear heads.

## **Shear Head To Improve Strength Against Punching Shear:**

Four types of shear heads, shown in Figs. 12.24 and 12.25, may be designed over the top of columns in case the concrete strength alone is not sufficient to resist the applied punching shear.

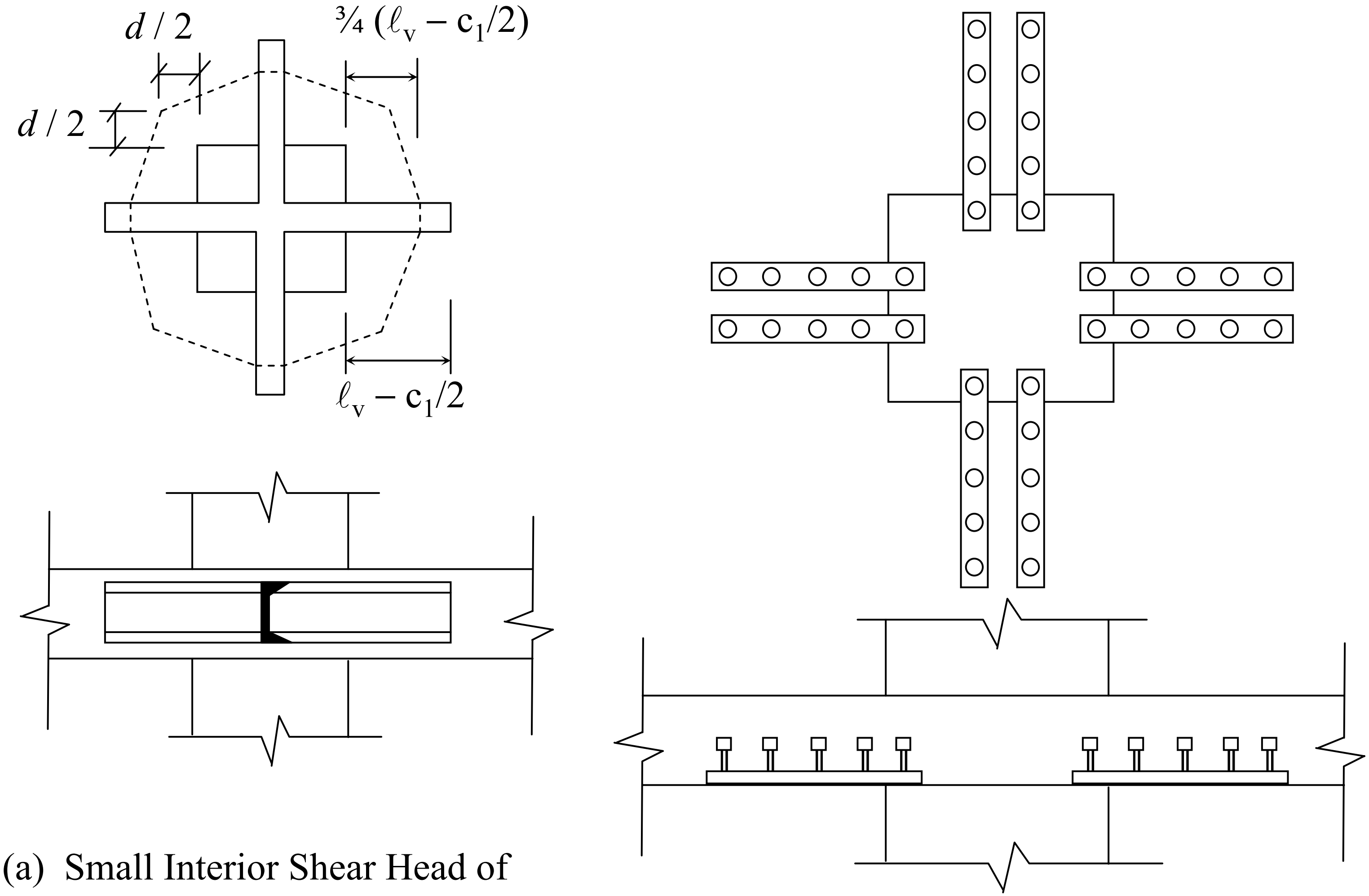



(a) Stirrup Shear Reinforcement.




(b) Large Interior Shear Head of Channel Sections.

Fig. 12.24. Typical Shear Heads.




(a) Small Interior Shear Head of Channel Section.

(b) Shear Head of Stud Connectors.

Fig. 12.25. More Examples of Shear Heads.

According to ACI 11.12.3, shear reinforcement consisting of bars, wires, single leg stirrups and double leg stirrups may be provided in slabs and footings with effective depth greater than or equal to greater of 150 mm and 16 times the shear reinforcement bar diameter.

This reinforcement must engage the longitudinal flexural reinforcement in the direction being considered.

To calculate the shear strength with shear reinforcement, the maximum value of $V_c$ is taken equal to $1/6\sqrt{f_c'}b_o d$ and $V_s$ is not to be taken greater than $1/3\sqrt{f_c'}b_o d$.

In other words, the maximum two-way shear strength cannot exceed $1/2\sqrt{f_c'}\,b_o d$, even if shear reinforcement is provided.

The area of shear reinforcement, $A_v$, is equal to area of all legs of reinforcement on one perimeter of the column section.

- The distance between the column face and the first line of stirrup legs that surround the column must not exceed $d/2$.
- The spacing parallel to the column face between the stirrups in this first line must not exceed $2d$.
- The spacing between successive lines of shear reinforcement that surround the column must not exceed $d/2$ measured in a direction perpendicular to the column face.

- Structural steel I– and channel–shaped sections are also allowed in the slabs.
- Arms of the shear-head must not be interrupted within the column sections.

- The section should not be a depth greater than 70 times the web thickness of the steel shape.
- All compression flanges of the structural shapes are to be located within 0.3$d$ of compression surface of the slab and these sections may be considered to be effective in resisting the moments besides providing the shear strength.
- The ratio ($a_v$) between the flexural stiffness of each shear-head arm and that of the surrounding composite cracked slab section of width ($c_2 + d$) must not be less than 0.15.

**Example 12.1:** Perform check for punching shear of a two-way slab system (Fig. 12.26) at the given edge column. The panel size is 6m × 8m and all conditions of direct design method are satisfied. The other related data is as under:

$q_u$ = 11,000 Pa

$M_u$ (unbalanced) = 200 kN-m

$f_c'$ = 25 MPa

$h$ = 230 mm

$d$ = 190 mm

# Solution:

$\beta$ = longer / shorter sides ratio for the column = 2.0

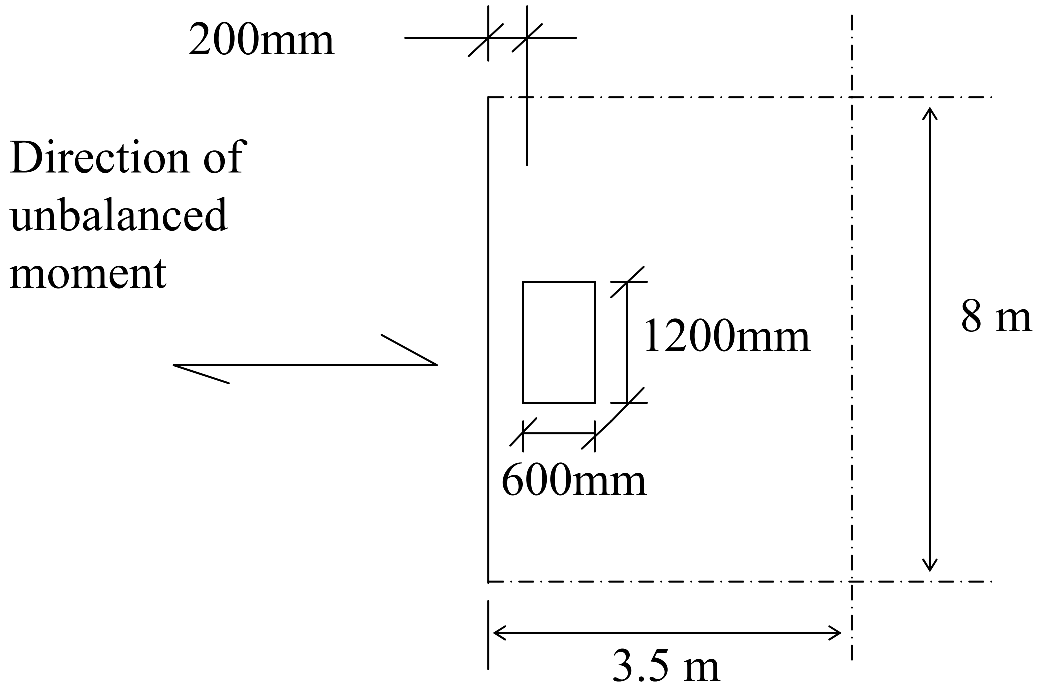



Fig. 12.26. Slab System For Example 12.1.

$b_1 = 600 + 190/2 + 200 = 895$ mm

$b_2 = 1200 + 190 = 1390$ mm

$b_o = 2\,b_1 + b_2 = 3180$ mm

$\alpha_s = 30$ for the edge column

$v_c$ = the least out of the following

i) $$\frac{1}{6}\left(1+\frac{2}{\beta}\right)\sqrt{f_c'} = \frac{1}{6}\left(1+\frac{2}{2.0}\right)\sqrt{25} = 1.667 \text{ MPa}$$

ii) $$\frac{1}{12}\left(\frac{\alpha_s d}{b_o}+2\right)\sqrt{f_c'} = \frac{1}{12}\left(\frac{30\times 190}{3180}+2\right)\sqrt{25} = 1.580 \text{ MPa}$$

iii) $\frac{1}{3}\sqrt{f_c'} = \frac{1}{3}\sqrt{25} = 1.667 \text{ MPa}$

$\phi v_c = 0.75 \times 1.580 = 1.185 \text{ MPa}$

$A_c = b_o \times d = 3180 \times 190 = 604{,}200 \text{ mm}^2$

$x_1 = \frac{b_1^2}{2b_1 + b_2} = \frac{895^2}{2 \times 895 + 1390} = 252 \text{ mm}$

$$J_c = 2 \times \left\{ \frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + b_1 d \left( \frac{b_1}{2} - x_1 \right)^2 \right\} + (b_2 d)(x_1)^2$$

$$= 2 \times \left\{ \frac{895 \times 190^3}{12} + \frac{190 \times 895^3}{12} + 895 \times 190 \times \left( \frac{895}{2} - 252 \right)^2 \right\}$$

$$+ (1350 \times 190)(252)^2$$

$$J_c = 5{,}349{,}563 \times 10^4 \text{ mm}^4$$

$$\gamma_f = \frac{1}{1+\frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = \frac{1}{1+\frac{2}{3}\sqrt{\frac{895}{1390}}} = 0.651$$

$$\gamma_f = 1 - \gamma_f = 0.349$$

More flexural steel is to be provided near the column, in a width of $c_2 + 3h$, to transfer 65.1% of moment.

Direct shear force, $V_u = q_u\,[\text{length} \times \text{width} - b_1\, b_2]$

$$= \frac{11{,}000}{1000}\left[(3.5)(8)-(0.895)(1.390)\right] = 294.3 \text{ kN}$$

Direct shear stress = $V_u / A_c$ = $294.3 \times 1000 / 604{,}200$ = 0.487 MPa

Eccentric shear = $\dfrac{\gamma_v M_u x_1}{J_c}$

$$= \frac{0.349 \times 200 \times 10^6 \times 252}{5{,}349{,}563 \times 10^4} = 0.329 \text{ MPa}$$

Total applied shear, $v_u$ = $0.486 + 0.329 = 0.815\ MPa$

$v_u \leq \phi v_c \Rightarrow$ The slab is safe against two-way shear.

If $v_u > \phi v_c$, following solutions are possible:

- Design a shear head.
- Provide drop panels or column capitals.
- Slightly increase the depth of slab if the difference of $v_u$ and $\phi v_c$ is smaller.

**Example 12.2:** Design reinforcement for the interior panel of the flat plate floor system, shown in Fig. 12.27. Assume that direct design method is applicable and the depth criterion is satisfied using a depth of 220 mm. Check the depth of slab for shear considering the effect of eccentric shear equal to 15% of the direct shear for this interior panel. The other related data is as under:

| | | |
|---|---|---|
| Live load | = | 300 $kgs/m^2$ |
| Floor finish and partitions | = | 150 $kgs/m^2$ |
| $M_u$ (unbalanced) | = | 200 kN-m |
| $f_c'$ | = | 25 MPa |
| $f_y$ | = | 300 MPa |
| $h$ | = | 220 mm |

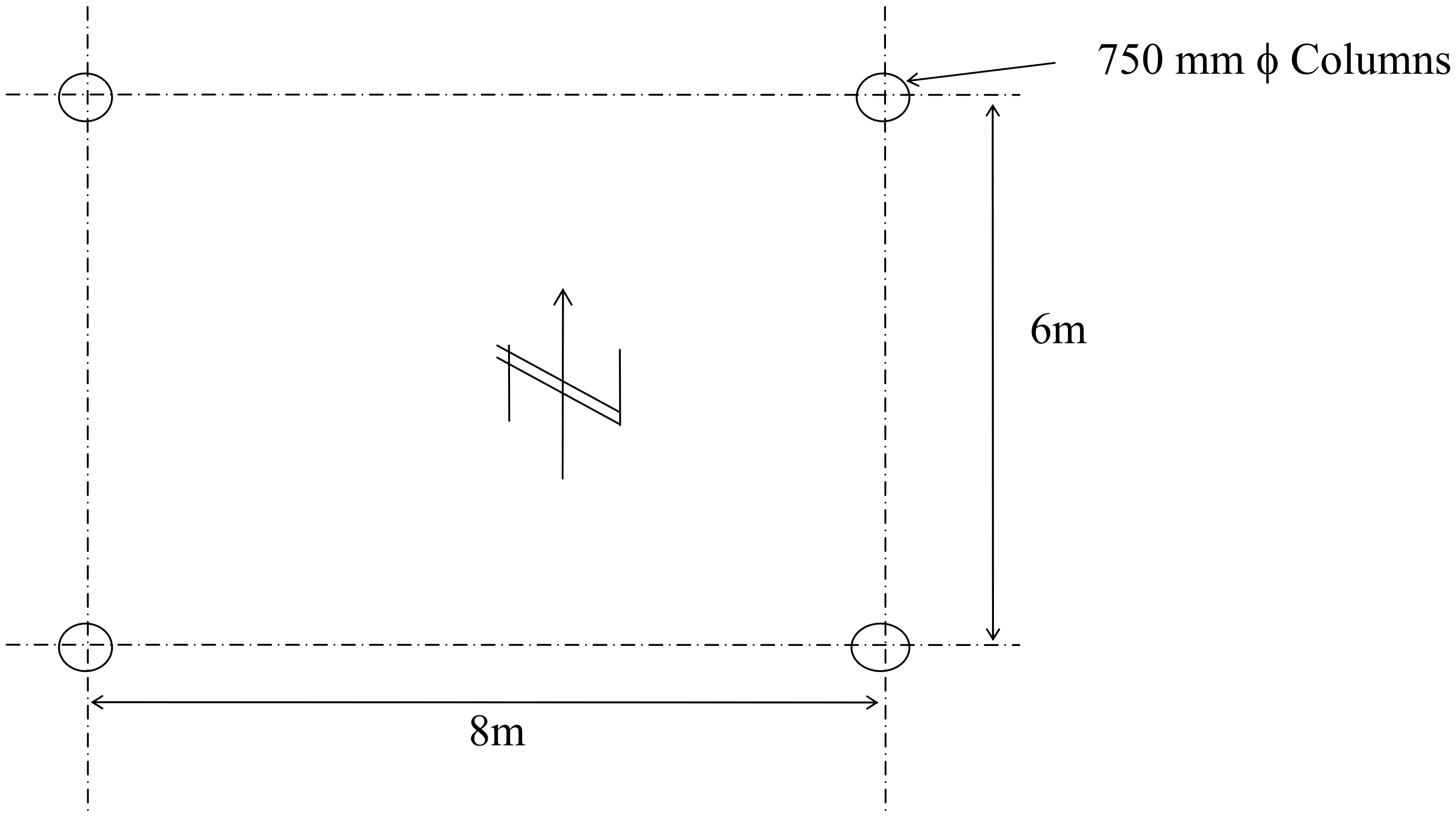



Fig. 12.27. Typical Interior Panel of Slab System For Example 12.2.

**Solution:**

Equivalent square column side, $h = \sqrt{\frac{\pi}{4}}(750) = 665$ mm

$$
\begin{aligned}
q_L &= 300 \text{ N/m}^2 \\
q_D &= 0.220 \times 2400 + 150 = 678 \text{ N/m}^2 \\
q_u &= [1.2(q_D) + 1.6(q_L)] \times 9.81 / 1000 \\
&= [1.2(678) + 1.6(300)] \times 9.81 / 1000 \\
&= 12.69 \text{ kN/m}^2
\end{aligned}
$$

The column and middle strips are shown in Fig. 12.28.

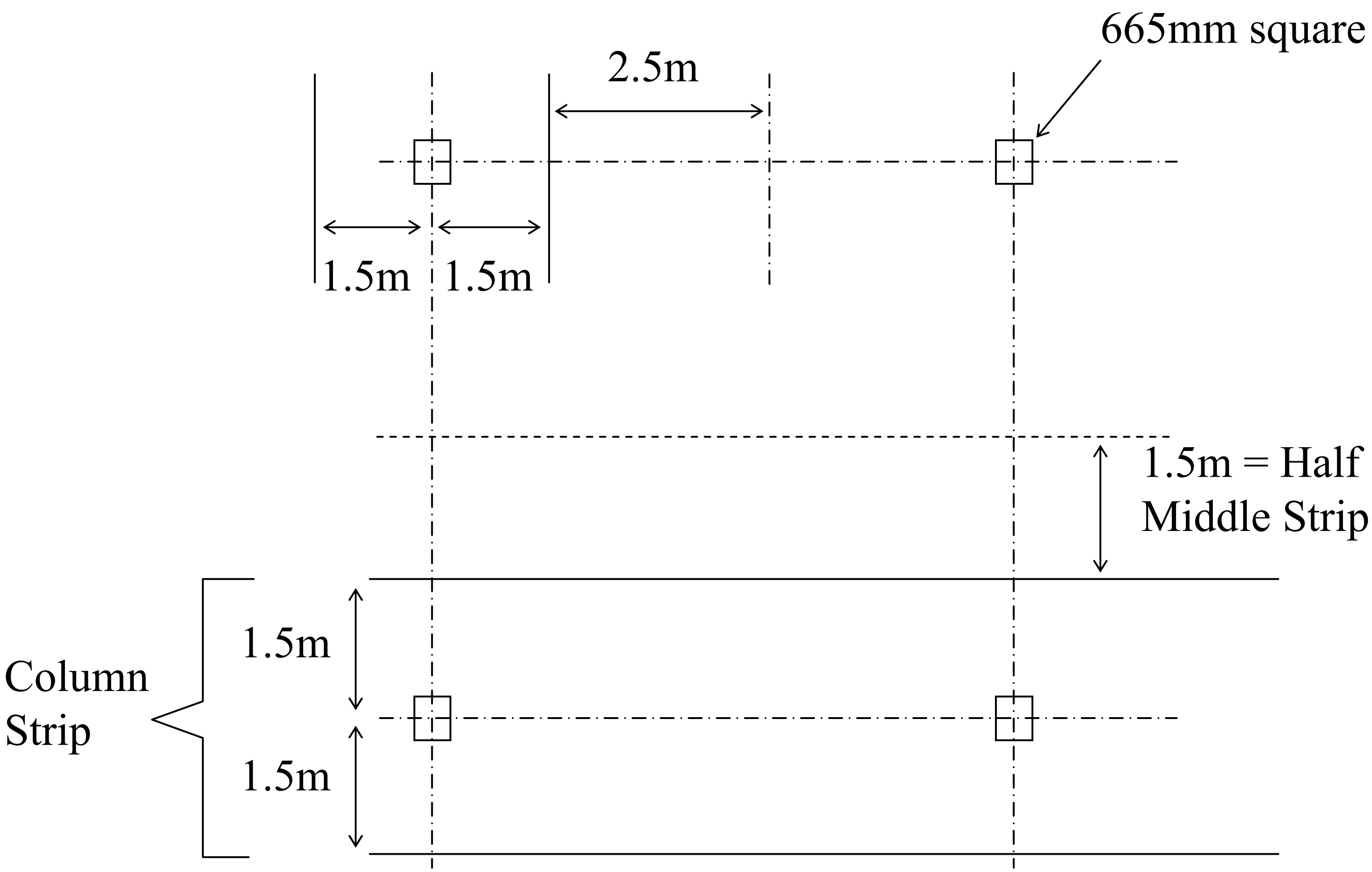



Fig. 12.28. Equivalent Columns And Column And Middle Strips.

1. E–W Span

$\ell_1 \quad = \quad 8\text{m}$

$\ell_2 \quad = \quad 6\text{m}$

$\ell_n \quad = \quad 8.0 - 0.665 \;=\; 7.335\text{ m}$

$\ell_{2w} \quad = \quad 6.0\text{ m}$

$$M_o \quad = \quad q_u \frac{\ell_{2w}\,\ell_n^2}{8} \;=\; (12.69)\frac{(6.0)(7.335)^2}{8} = \;512.1\text{ kN-m}$$

***Support Section (Top steel, E–W Direction)***

$M^- \;=\; 0.65\,M\text{o} \;=\; 0.65\,(512.1) \;=\; 332.9\text{ kN-m}$

$A = \ell_2/\ell_1 = 0.75$

$\alpha_{f1} = 0$ (no beam), $D \;=\; 0$

%age CS moment out of positive moment = 75%

%age CS moment out of positive moment = 60%

Column Strip: $0.75\ (332.9) =$ 249.7 kN-m
Middle Strip: $0.25\ (332.9) =$ 83.2 kN-m

***Mid Span Section (Bottom steel, E–W Direction)***

$M^{+} = 0.35\ M_o = 0.35\ (512.1) =$ 179.2 kN-m
Column Strip $= 0.60\ (179.2) =$ 107.6 kN-m
Middle Strip $= 0.40\ (179.2) =$ 71.6 kN-m

## 2.0 N – S Span

$\ell_1 = 6m$
$\ell_2 = 8m$
$\ell_n = 6.0 - 0.665 = 5.335m$
$\ell_{2w} = 8m$

$$M_o = (12.69)\ \frac{(8.0)(5.335)^2}{8} = 361.2 \text{ kN-m}$$

***Support Section (Top steel, N–S Direction)***

$M^- = 0.65\, M_o = 0.65\,(361.2)$

$= 234.8$ kN-m

Column Strip $= 0.75\, M^- = 0.75\,(234.8) = 176.1$ kN-m

Middle Strip $= 0.25\, M^- = 0.25\,(234.8) = 58.7$ kN-m

***Mid Span Section***

$M^+ = 0.35\, M_o = 0.35\,(361.2) = 126.4$ kN-m

Column Strip $= 0.60\,(126.4) = 75.8$ kN-m

Middle Strip $= 0.40\,(126.4) = 50.6$ kN-m

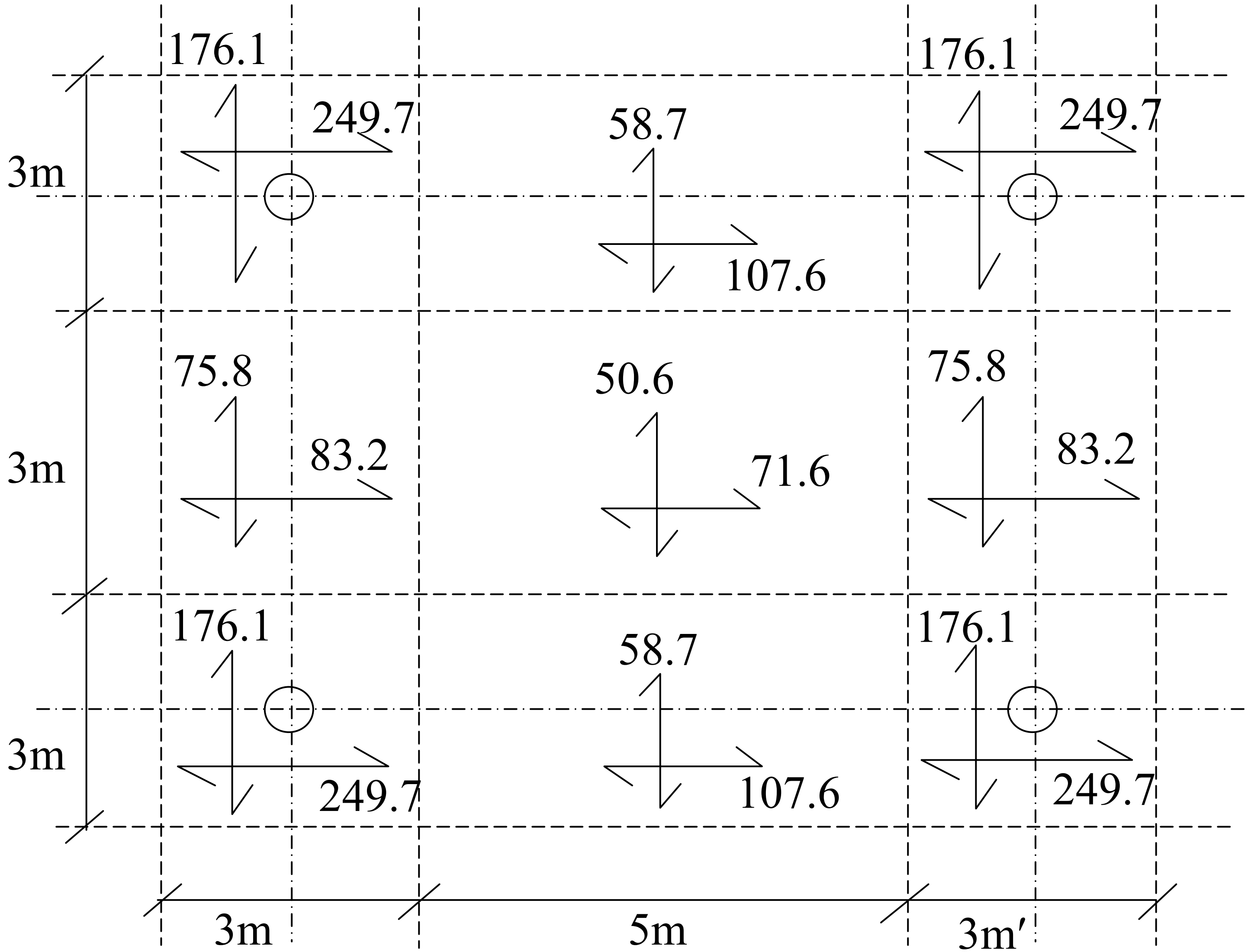



Fig. 12.29. Moments Across Full Width of Strips For Example 12.2.

$$s_{\max} = 2h = 440 \text{ mm}$$

$$d = 220 - 20 - 16 - 6 = 178 \text{ mm}$$

(lesser value for the inner steel)

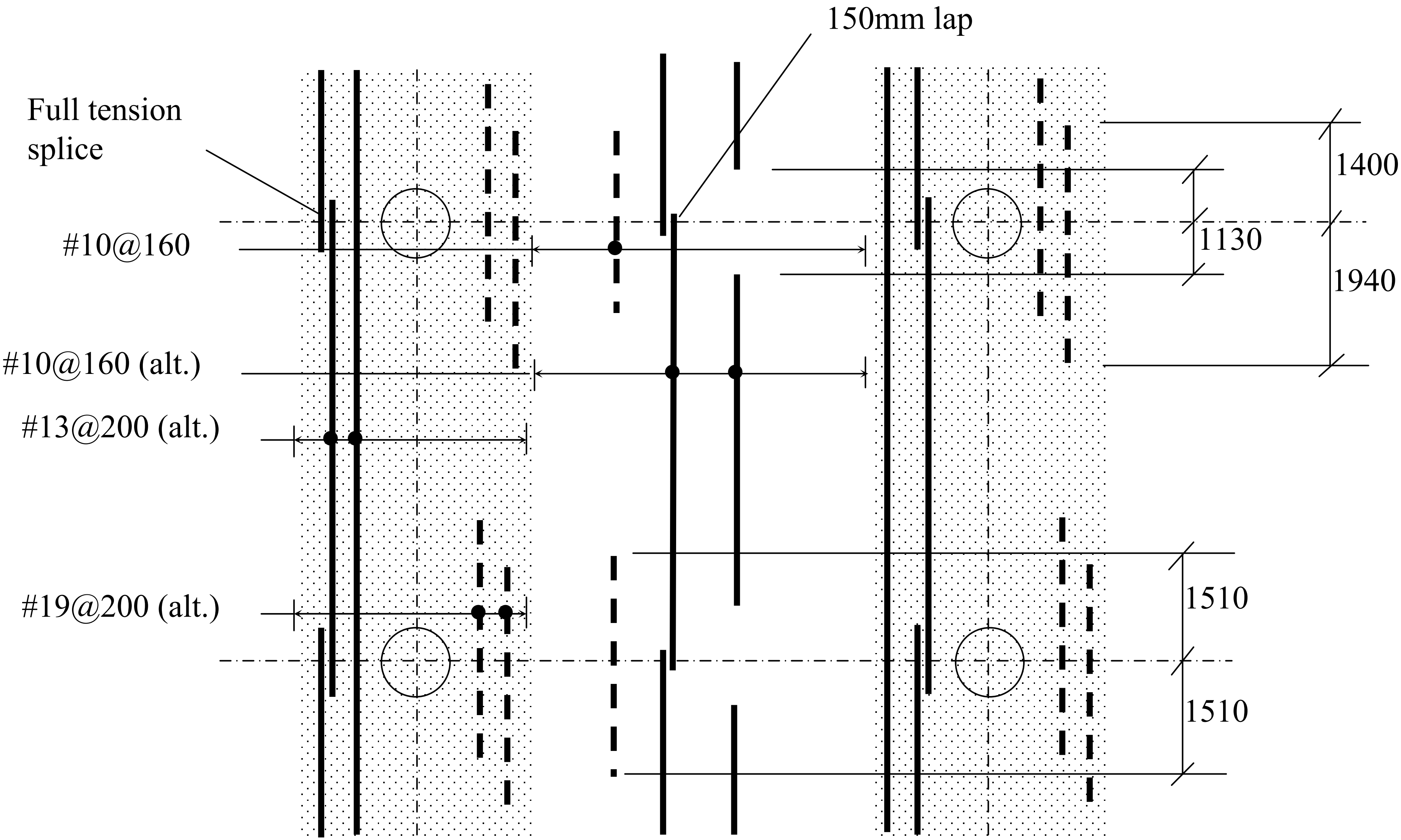
$$\rho_{\min} = 0.0020, \quad A_{s,\min} = 0.002 \times 1000 \times 220$$

$$= 440 \text{ mm}^2/\text{m width}$$

| Table 12.12. Calculation of Slab Steel For Example 12.2. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Design Frame** | **Location** | **Strip** | **Width mm** | **$M_u$ kN-m** | **R $=M_u/bd^2$** | $\rho$ | **As $mm^2$** | **Steel** |
| E-W | Support Top steel | CS | 3000 | 249.7 | 2.627 | 0.0106 | 1887 | #19@150mm c/c |
| | | MS | 3000 | 83.2 | 0.875 | 0.0033 | 587 | #13@200mm c/c |
| E-W | Mid-span Bot. steel | CS | 3000 | 107.6 | 1.132 | 0.0046 | 819 | #13@150mm c/c |
| | | MS | 3000 | 71.6 | 0.753 | 0.0029 | 516 | #13@250mm c/c |
| N-S | Support Top steel | CS | 3000 | 176.1 | 1.853 | 0.0076 | 1353 | #19@200mm c/c |
| | | MS | 5000 | 58.7 | 0.371 | 0.0025 | 445 | #10@160mm c/c |
| N-S | Mid-span Bot. steel | CS | 3000 | 75.8 | 0.797 | 0.0033 | 587 | #13@200mm c/c |
| | | MS | 5000 | 50.6 | 0.319 | 0.0025 | 445 | #10@160mm c/c |

Table 12.13. Curtailment of Slab Steel For Example 12.2.

| **Design Frame** | **Span Lengths** | | **Column Strip + ½ Eq. Column Size** | | **Middle Strip + ½ Eq. Column Size** | |
|---|---|---|---|---|---|---|
| | $\ell_1$ (mm) | $\ell_n$ (mm) | 0.30 $\ell_n$ (mm) | 0.20 $\ell_n$ (mm) | 0.22 $\ell_n$ (mm) | 0.15 $\ell_n$ (mm) |
| E-W | 8000 | 7335 | 2540 | 1800 | 1950 | 1430 |
| N-S | 6000 | 5335 | 1940 | 1400 | 1510 | 1130 |


150mm lap
Full tension splice
#10@160
#10@160 (alt.)
#13@200 (alt.)
#19@200 (alt.)
1400
1130
1940
1510
1510

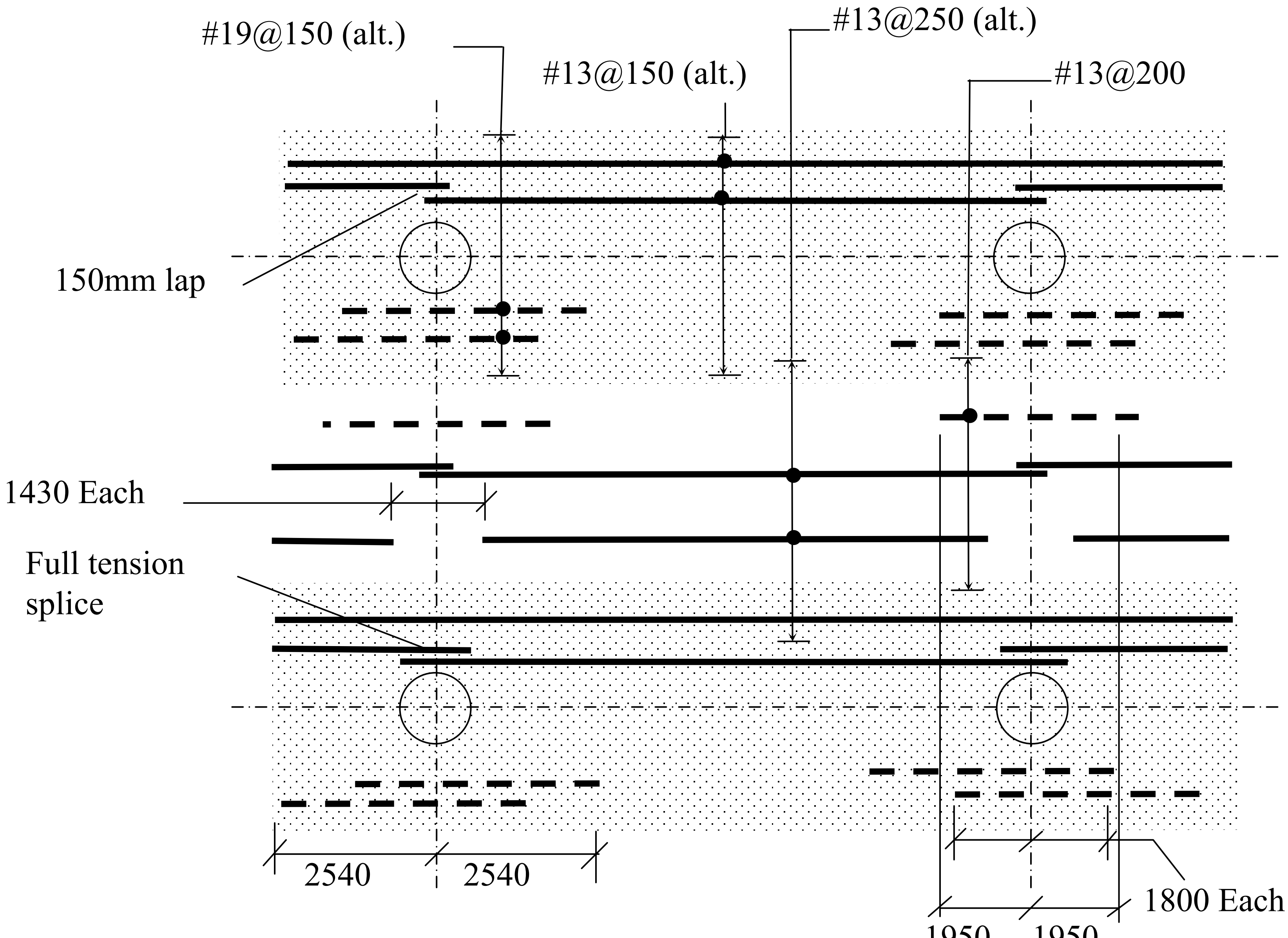



Fig. 12.30. Detailing For Slab of Example 12.2.

**Example 12.3:** Calculate the design moments for the exterior panel of the flat plate system given in Fig. 12.31, perpendicular to the edge. The other related data is as under:

| | | |
|---|---|---|
| Clear cover | = | 20 mm |
| Grade of steel | = | 420 MPa |
| Superimposed $q_D$ | = | 150 kgs/m$^2$ |
| Live load $q_L$ | = | 300 kgs/m$^2$ |
| $f_c'$ | = | 20 MPa |

<u>**Solution:**</u>

Longer $\ell_n$ = (6000 – 375) / 1000 = 5.625 m

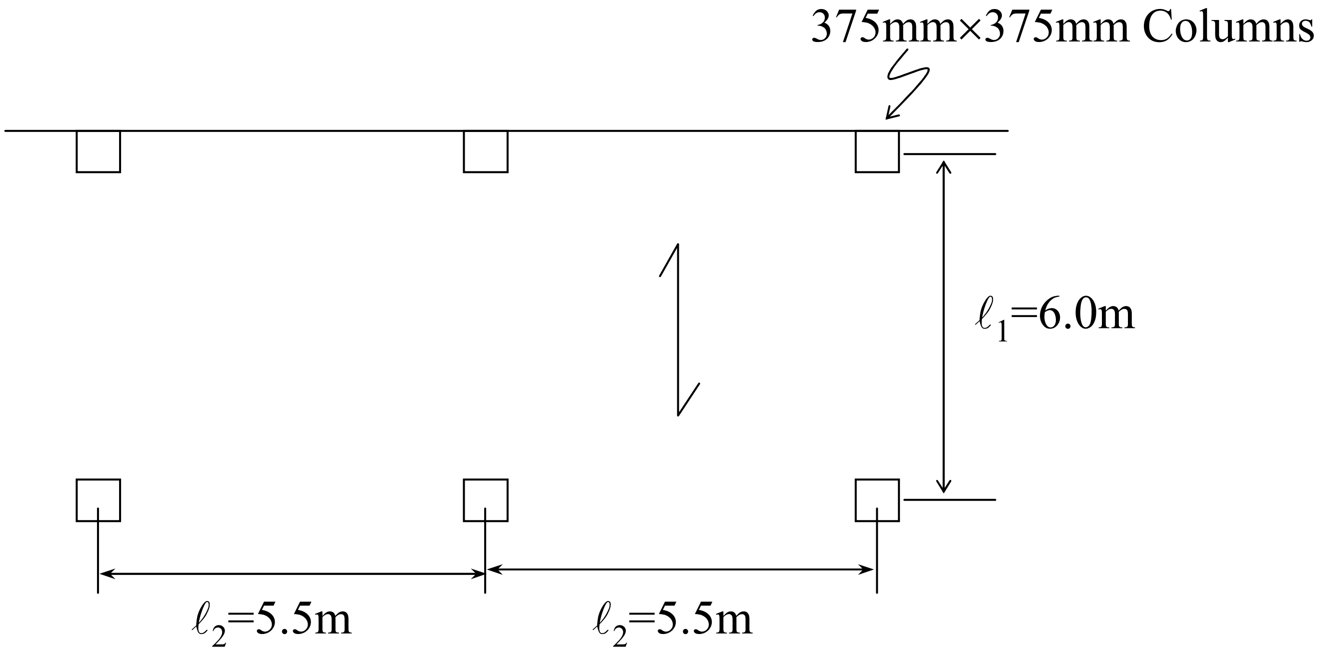



Fig. 12.31. Flat Slab of Example 12.3.

$$h_{min.} \cong \ell_n / 30 \text{ for } f_y = 420 \text{ MPa} \geq 120 \text{ mm}$$

$$= (5.625 \times 1000) / 30 = 187.5 \text{ mm} \geq 120 \text{ mm}$$

***OK***

Try $h = 200$ mm

## Total Static Moment

$$q_D = 0.2 \times 2400 + 150 = 630 \text{ kgs/m}^2$$

$$q_L = 300 \text{ kgs/m}^2$$

$$q_u = [1.2(630) + 1.6(300)] \times 9.81 / 1000$$

$$= 12.13 \text{ kN/m}^2$$

$$\ell_{2w} = 5.5 \text{ m}$$

$$M_o = \frac{q_u \ell_{2w} \ell_n^2}{8} = \frac{12.13 \times 5.5 \times 5.625^2}{8}$$

$$= 263.8 \text{ kN-m}$$

## Longitudinal Distribution Of Moments

$M_o = 263.8$

- Int. $M^{-} = 0.70M_o = -184.7$ kN-m
- $M^{+} = 0.52M_o = +137.2$ kN-m
- Ext. $M^{-} = 0.26M_o = -68.6$ kN-m

## Torsional Member

There is no edge beam and 375mm width of slab may be assumed to act as a torsion member, as shown in Fig. 12.32.

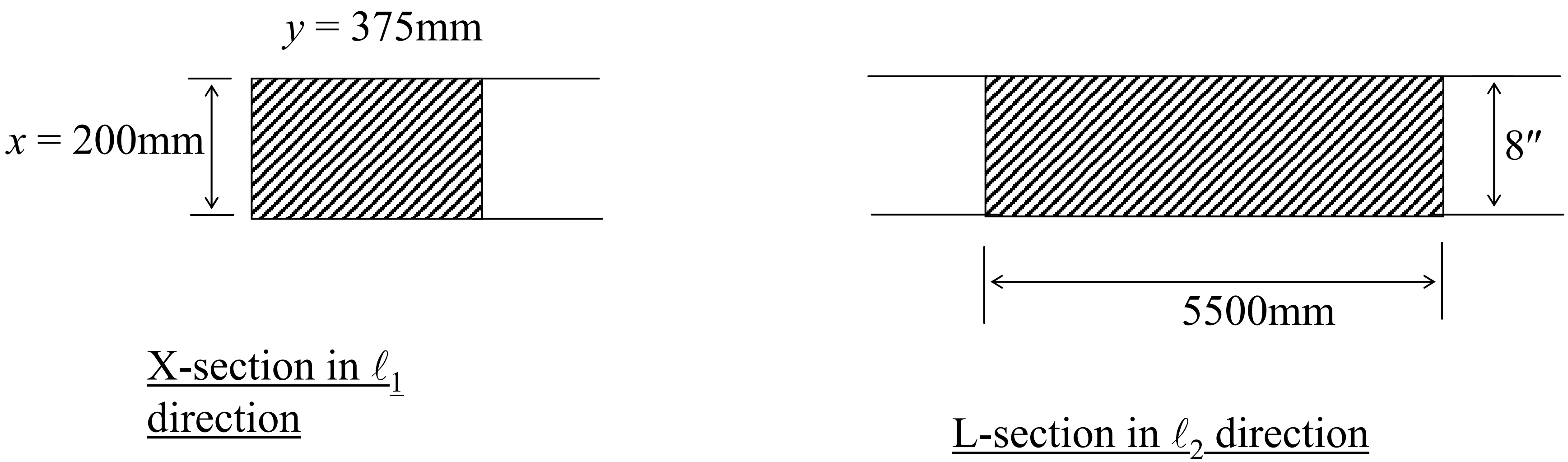



Fig. 12.32. Torsion Member For Example 12.3.

$$C = \sum\left(1-0.63\frac{x}{y}\right)\frac{x^3 y}{3} = 66{,}400 \times 10^4 \text{ mm}^4$$

$$I_s = \frac{5500\times 200^3}{12} = 366{,}667 \times 10^4 \text{ mm}^4$$

$$\beta_t = \frac{E_{cb}\,C}{2\,E_{cs}\,I_s}$$

$$= \frac{66{,}400 \times 10^4}{(2)(366{,}667 \times 10^4)} = 0.09$$

$$A = \ell_2 / \ell_1 = 5.5 / 6.0 = 0.917$$

$$B = \beta_t = 0.09$$

$$\alpha_{f1} = 0 \Rightarrow D = \alpha_{f1}\frac{\ell_2}{\ell_1} = 0$$

**Transverse Distribution Of Moments**

The column strip moment percentages are calculated as under:

$$\text{Int. } M^- = 75 + 30(1-A)D = 75\%$$

$$\text{Ext. } M^- = 100 - 10B + 12BD(1 - A) = 99.1\%$$

$$M^+ = 60 + 15(3-2A)D = 60\%$$

$M_o = 263.8$

- Int. $M^- = 0.70M_o = -184.7$ kN-m
  - CS = 0.75(–184.7) = 138.5kN-m
  - MS = 0.25(–184.7) = 46.2kN-m
- $M^+ = 0.52M_o = +137.2$ kN-m
  - CS = 0.60(137.2) = 82.3kN-m
  - MS = 137.2–82.3 = 54.9kN-m
- Ext $M^- = 0.26M_o = -68.6$ kN-m
  - CS = –68.0k-ft (99.1%)
  - MS = very small

Continued on next file