EQUIVALENT FRAME METHOD

- Equivalent frame method (EFM) is described in ACI 13.7.

- It is a general method for design of two-way column supported slab systems, without the restrictions of the direct design method.

- However, this method is only applicable in case of gravity loads and all general provisions for two-way slabs, except those of ACI 13.6, are also applied in this method.

- The three-dimensional slab systems are first divided into two-dimensional design frames by cutting at the panel centerlines.

- The removal of the torsional links between various design frames makes this method conservative in nature.

- The longitudinal distribution of moments for these design frames is carried out by performing actual 2-D frame analysis.

- This requires some modifications before implementation.

- Firstly, equivalent column stiffness is to be calculated combining the effects of actual column stiffness, unsupported edge of slab and the torsion member.

- Secondly, the variation of moment of inertia of the horizontal member along its length between the column centerlines is to be considered.

- Thirdly, the variation of moment of inertia of the column between the centerlines of horizontal members must be considered.

- The horizontal member in the equivalent frame consisting of slab, beams (if present) and drop panels (if present) is termed ***slab-beam***.

- There are four main steps involved in the use of the equivalent frame method.

- *Step 1:* The 3–D slab system is represented by four or more 2–D frames, just like in DDM.

  These design strips (or design frames) are separately considered for analysis and design.

- *Step 2:* The stiffness of frame elements is determined considering the facts that the slab is not supported along full width at the edge, torsion member is present but its effect can not be included directly in the 2-d analysis and the columns and the slab-beam are non-prismatic members.

Torsional stiffness of transverse beams and the slab edge condition at the junction with the outer column are included in the column stiffness and thus the concept of an equivalent column is used.

ACI Code allows the analysis of a particular floor of the building by considering a free body of that floor with the columns below and the columns above (if present), with the far ends of these columns taken as fixed.

This simplification is very useful for hand calculations.

- ***Step 3:*** The 2-D frames obtained in the above step are analyzed for full gravity loads (pattern loading is not considered if live load is within certain percentage of the dead load).

  This is equivalent to the longitudinal distribution of total static moment in the direct design method.

- ***Step 4:*** The negative and positive moments (M^- and M^+) are distributed laterally to column strips and middle strips using coefficients of DDM if the following limitation is satisfied:

$$0.2 \leq \frac{\alpha_{f1}\ell_2^2}{\alpha_{f2}\ell_1^2} \leq 5.0$$

Figure 12.36 graphically represents the variation of moment of inertia along the length of the slab-beams.

For hand calculations, the non-prismatic slab-beam member may approximately be converted into a prismatic member by modifying the fixed end moments, flexural stiffness and carryover factors as under:

$$\text{FEM} = m \times q\, \ell_{2w}\, \ell_1^2$$
$$K_{sb} = k \times EI_{sb}\, /\, \ell_1$$

$$I_3 = \frac{I_2}{\left(1 - c_2/\ell_2\right)^2}$$

I_2

I_1

Variation of Moment of Inertia

$$I_3 = \frac{I_2}{\left(1 - c_2/\ell_2\right)^2}$$
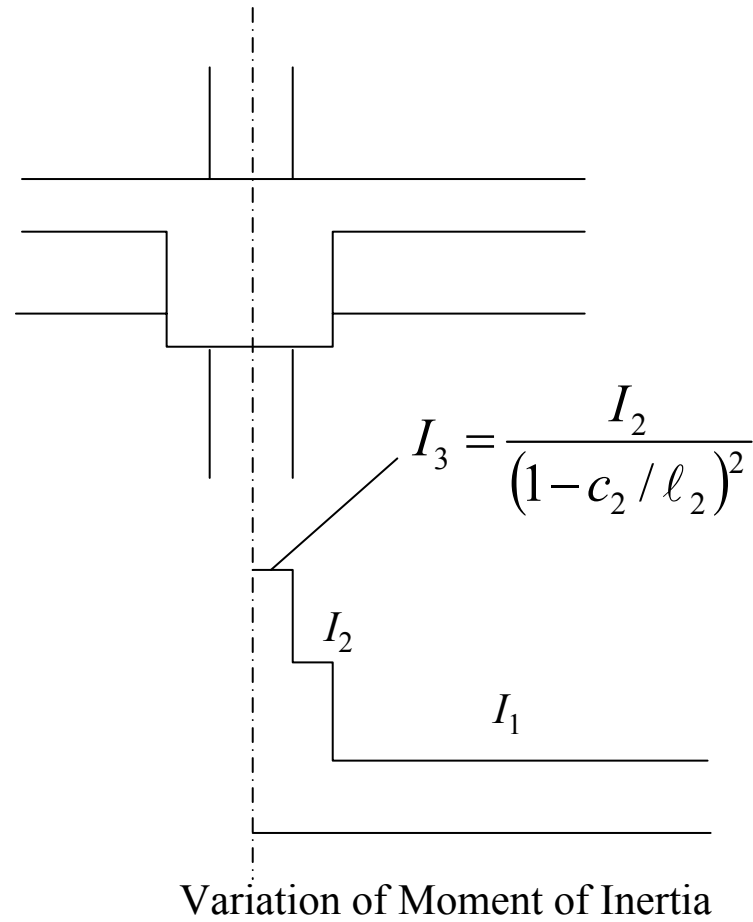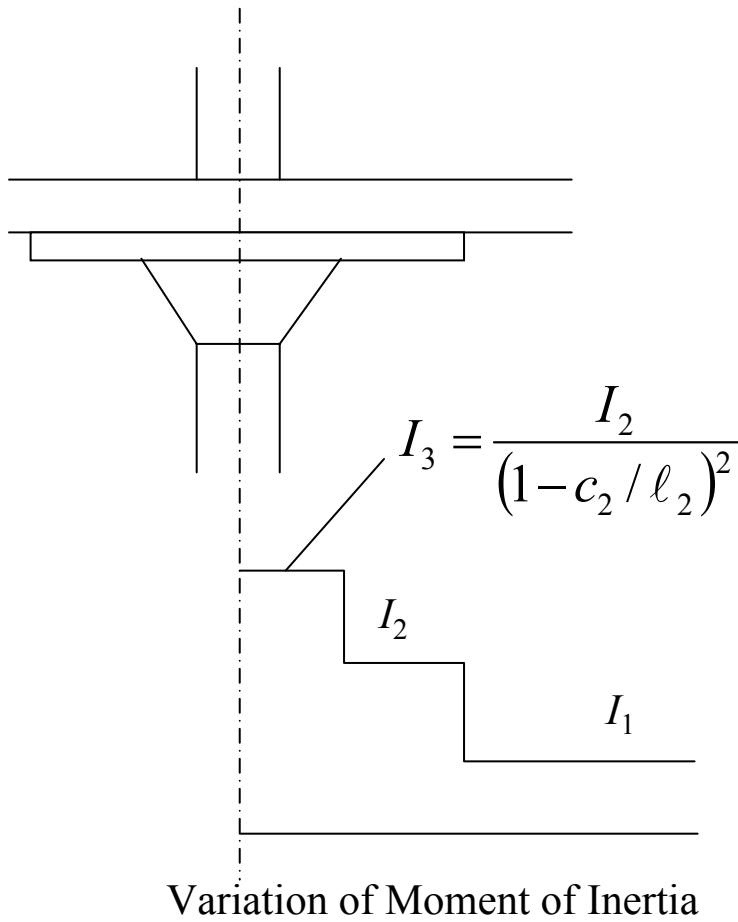
I_2

I_1

Variation of Moment of Inertia

Fig. 12.36. Stiffness Variation in Case of Slab-Beams.

$$\alpha \quad = \quad \frac{\text{depth at drop panel}}{\text{depth of slab}}$$

c_1 = larger of column or capital width at the top in the direction of calculation of moments.

c_2 = larger of column or capital width at the top perpendicular to the direction of calculation of moments.

$$m \quad = \quad 0.09 \left(\frac{c_1}{\ell_1} \times \frac{c_2}{\ell_2} \right)^{0.015} \alpha^{0.24} \qquad \geq 0.083$$
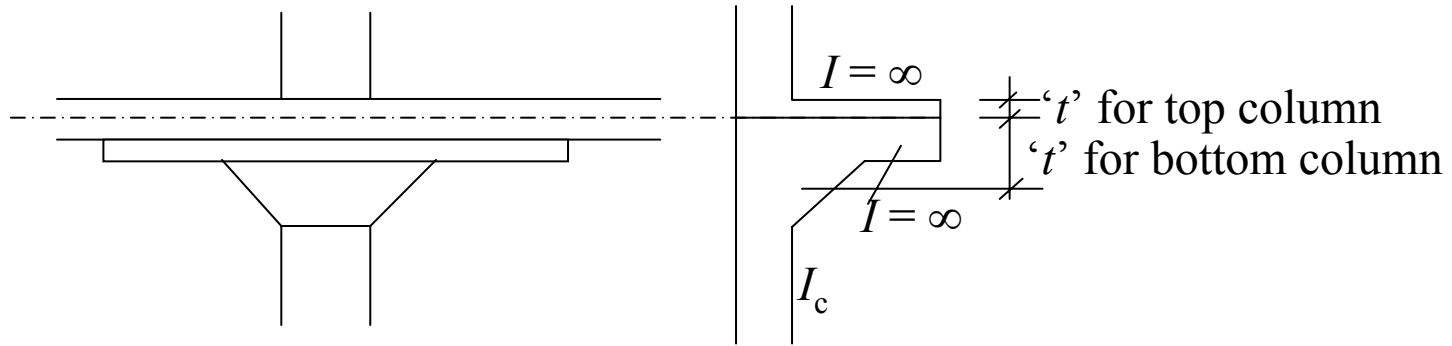
$$k \quad = \quad 5.3 \left(\frac{c_1}{\ell_1} \times \frac{c_2}{\ell_2} \right)^{0.05} \alpha^{0.9} \qquad \geq 4.0$$

$$COF = 0.57 \left(\frac{c_1}{\ell_1} \times \frac{c_2}{\ell_2} \right)^{0.02} \alpha^{0.37} \geq 0.5$$

Moment of inertia of a column from the top of slab-beam to its bottom is considered equal to infinity.

The variation of column stiffness along the height for some typical cases is shown in Fig. 12.37.

ℓ_c = c/c height of the columns

ℓ_u = unsupported height of column

     = $\ell_c - t_a - t_b$

t = vertical distance starting from the slab centerline up to inner end of slab, inner end of drop panel or mid-height of column capital (Fig. 12.37)

$I = \infty$

't' for top column
't' for bottom column

$I = \infty$

I_c

Variation of Moment
of Inertia

$I = \infty$

$I = \infty$

I_c
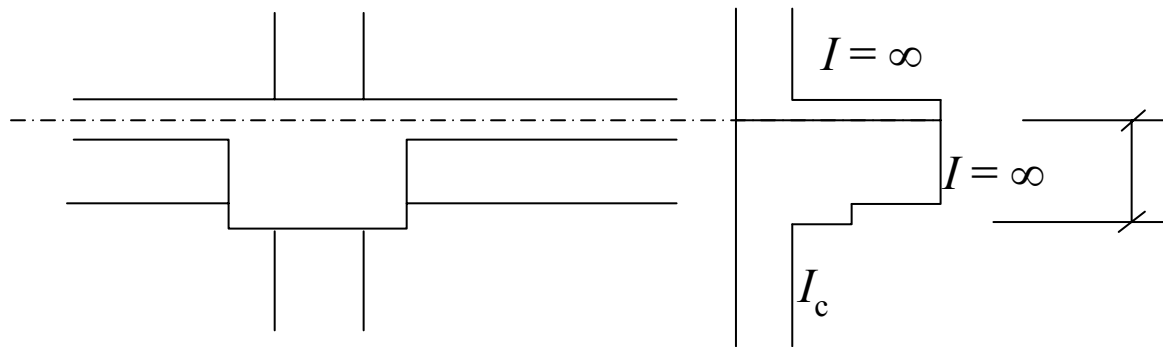
Variation of Moment
of Inertia

Fig. 12.37. Stiffness Variation of Columns Along Their Heights.

a-end = column end near the slab to be analyzed

b-end = column end away from the slab to be analyzed

t_a = t-value at a-end of column

t_b = t-value at b-end of column

$K_c,a = k_a \times EI_c / \ell_c$

$$k_a = 4.0 \left(\frac{t_a}{t_b}\right)^{0.08} \left(\frac{\ell_c}{\ell_u}\right)^{2.7} \geq 4.0$$

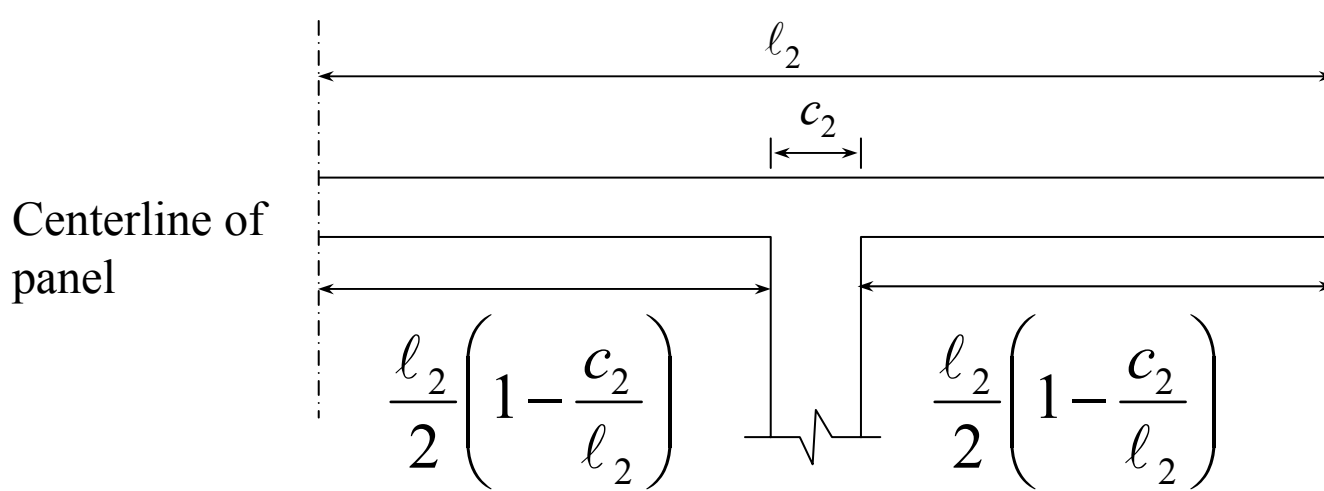for $t_a / t_b = 0.4$ to 2.2 and ℓ_c / ℓ_u up to 1.2

$$COF_a = 0.5 \ \frac{\left(\ell_c / \ell_u\right)}{\left(t_a / t_b\right)^{0.08}} \geq 0.5$$

Detailed 3–D analyses of various slab systems suggest that an approximate value of the torsional stiffness can be obtained by assuming a torque distribution along the torsional member that varies linearly from a maximum at the center of the column to zero at the middle of the panel, as shown in Fig. 12.38(a).
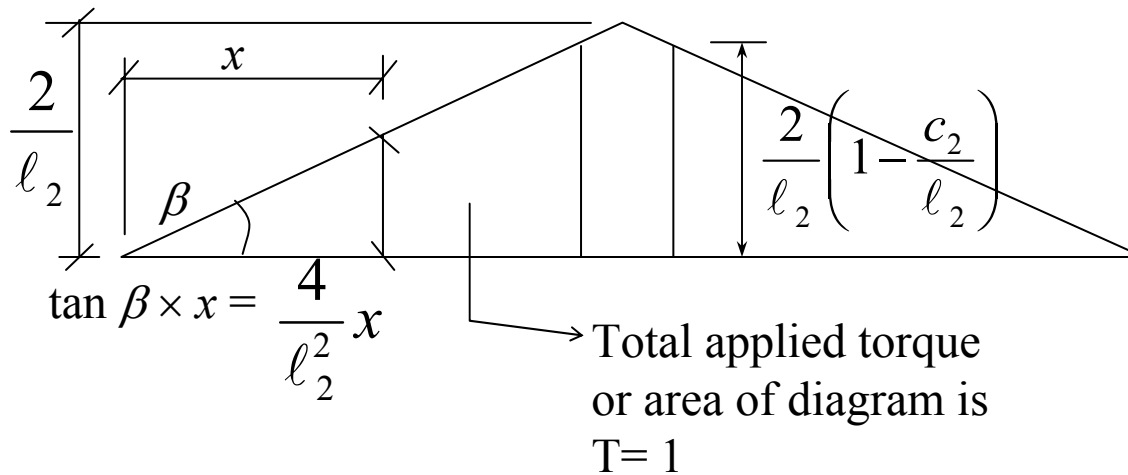
The distance from centerline of the panel to the edge of column is calculated as under:

Cantilever slab distance $= \ell_2 / 2 - c_2 / 2$

$$= \frac{\ell_2}{2}\left(1 - \frac{c_2}{2} \times \frac{2}{\ell_2}\right) = \frac{\ell_2}{2}\left(1 - \frac{c_2}{\ell_2}\right)$$
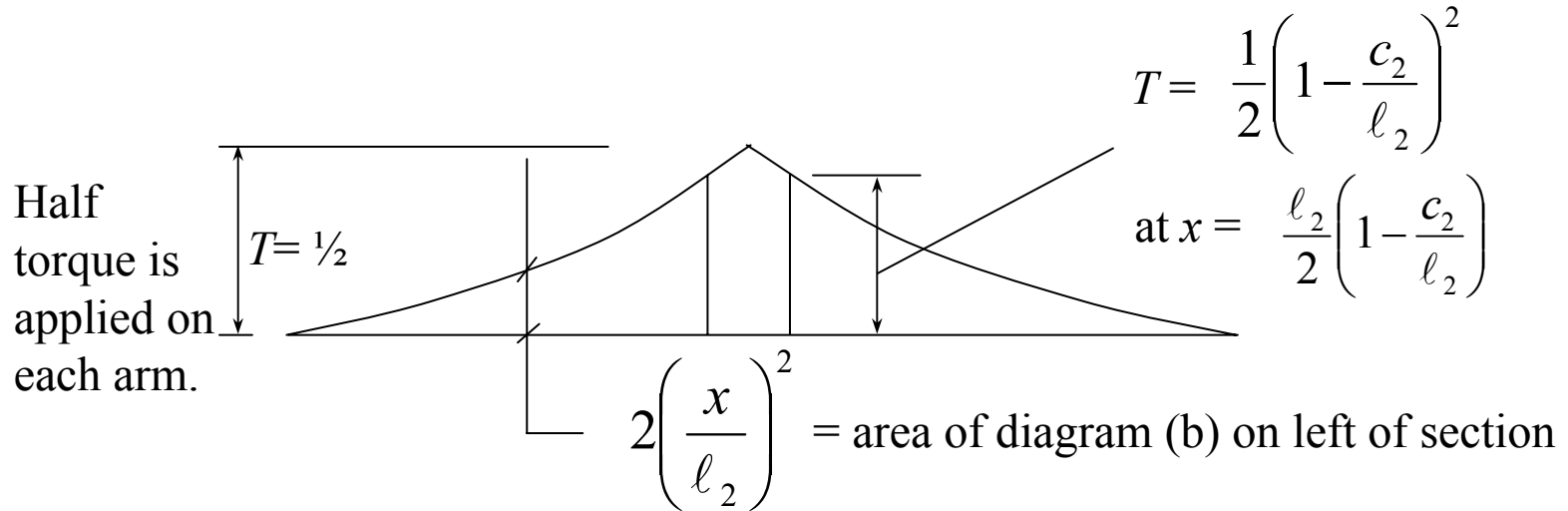
ℓ_2

c_2

Centerline of panel

$$\frac{\ell_2}{2}\left(1-\frac{c_2}{\ell_2}\right)$$

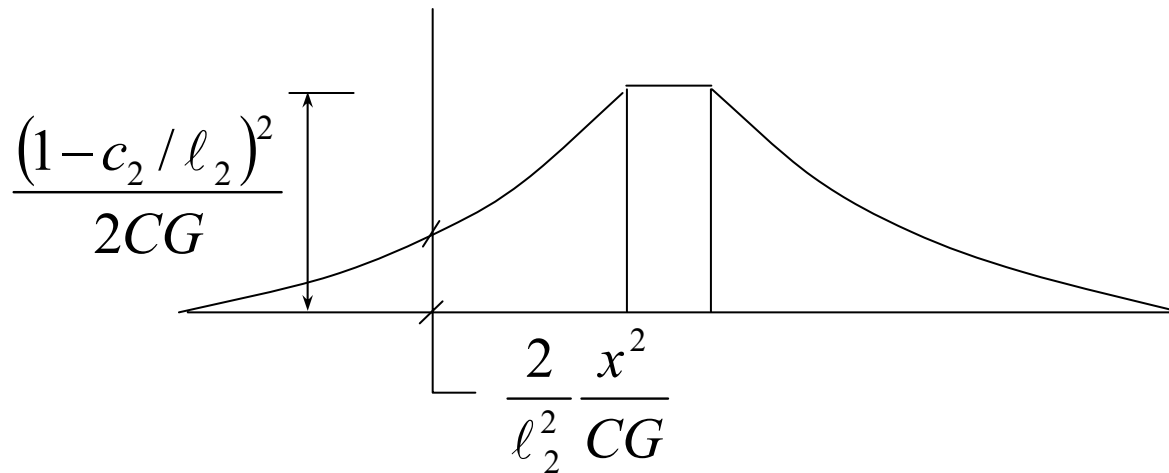$$\frac{\ell_2}{2}\left(1-\frac{c_2}{\ell_2}\right)$$

(a) Section of Design Strip

$$\tan\beta = \frac{2/\ell_2}{\ell_2/2} = \frac{4}{\ell_2^2}$$

x

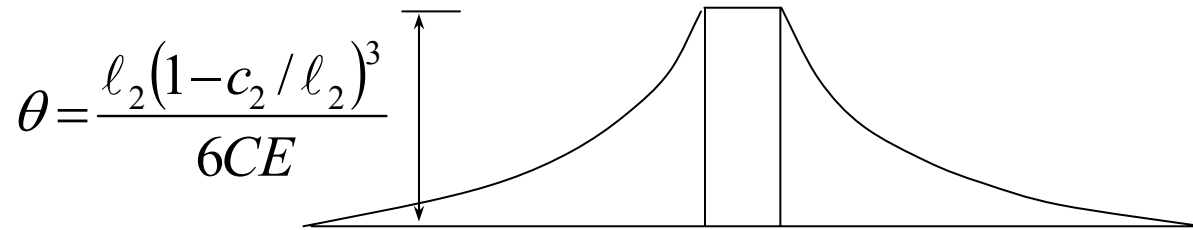$$\frac{2}{\ell_2}$$

$$\frac{2}{\ell_2}\left(1-\frac{c_2}{\ell_2}\right)$$

β

$$\tan\beta \times x = \frac{4}{\ell_2^2}x$$

Total applied torque or area of diagram is T= 1

(b) Torque Distribution Diagram

$$T = \frac{1}{2}\left(1 - \frac{c_2}{\ell_2}\right)^2$$

Half torque is applied on each arm.

$T = \frac{1}{2}$

$$\text{at } x = \frac{\ell_2}{2}\left(1 - \frac{c_2}{\ell_2}\right)$$

$$2\left(\frac{x}{\ell_2}\right)^2 = \text{area of diagram (b) on left of section}$$

(c) Resultant Torque At Each Section

$$\frac{\left(1 - c_2/\ell_2\right)^2}{2CG}$$

$$\frac{2}{\ell_2^2}\frac{x^2}{CG}$$

(d) Angle Change Per Unit Length

$$\theta = \frac{\ell_2\left(1 - c_2/\ell_2\right)^3}{6CE}$$

(e) Total Rotation At Sections Considering Free
End As The Reference Point

Fig. 12.38. Calculation of Slab Rotation Due to Applied Torque.

Resultant torque at distance x

= area of diagram-b from end up to distance x

$$= \quad \frac{1}{2} \times \frac{4x}{\ell_2^2} \times x \quad = \quad \frac{2x^2}{\ell_2^2}$$

Total rotation at distance x

$\quad = $ area of diagram-d in distance

$$= \int_{x}^{\frac{\ell_2}{2}\left(1-\frac{c_2}{\ell_2}\right)} \frac{2}{\ell_2^2 CG} x^2 dx$$

$$= \frac{2}{3\ell_2^2 CG}\left[\frac{\ell_2^3}{8}\left(1-\frac{c_2}{\ell_2}\right)^3 - x^3\right]$$

$$= \frac{\ell_2}{6CE}\left[\left(1-\frac{c_2}{\ell_2}\right)^3 - \left(\frac{2x}{\ell_2}\right)^3\right]$$

For poisson's ratio approximately considered equal to zero, $G = E / 2$. Considering $x = 0$, the total rotation of the free end with respect to the column may be calculated as follows:

$$\theta_{\text{end}} = \frac{\ell_2 \left(1 - c_2 / \ell_2\right)^3}{6\,CE}$$

The average rotation, θ_{av}, of the slab end may be found by evaluating the area of diagram-e and dividing by the length, ℓ_2.

$$\theta_{\text{av}} = \left[\frac{1}{4}\theta_{\text{end}}\frac{\ell_2}{2}\left(1 - \frac{c_2}{\ell_2}\right) \times 2 + c_2 \times \theta_{\text{end}}\right] / \ell_2$$

$$= \frac{\theta_{\text{end}}}{4}\left(1 + 3\frac{c_2}{\ell_2}\right)$$

$$= \frac{1}{4}\frac{\ell_2\left(1 - c_2/\ell_2\right)^3}{6\,CE}\left(1 + 3\frac{c_2}{\ell_2}\right)$$

$$\theta_{av} = \frac{\ell_2\left(1 - c_2/\ell_2\right)^3}{18\,CE}\left(\frac{3}{4} + \frac{9}{4}\frac{c_2}{\ell_2}\right)$$

$$= \frac{\ell_2\left(1 - c_2/\ell_2\right)^3}{18\,CE} \qquad \text{assuming } c_2 \approx \ell_2 / 9$$

Finally, the torsional stiffness of one arm is calculated as

$$K_t \text{ (one arm)} = \frac{T}{\theta_{av}} = \frac{\frac{1}{2} \times 18EC}{\ell_2\left(1 - c_2/\ell_2\right)^3} = \frac{9EC}{\ell_2\left(1 - c_2/\ell_2\right)^3}$$

In general, for cases where the arm is present on both sides, the total torsional stiffness is calculated as summation of the torsional stiffness for each arm.

$$K_t = \sum \frac{9E_{cs}\,C}{\ell_2\left(1 - c_2/\ell_2\right)^3}$$

According to ACI 13.7.5.2, where beams frame into columns in the direction of moments (1-direction), the torsional stiffness shall be multiplied by the ratio of moment of inertia of slab with such beam (I_{sb} in Fig. 12.39) to moment of inertia of slab without such beam (I_s in Fig. 12.39).
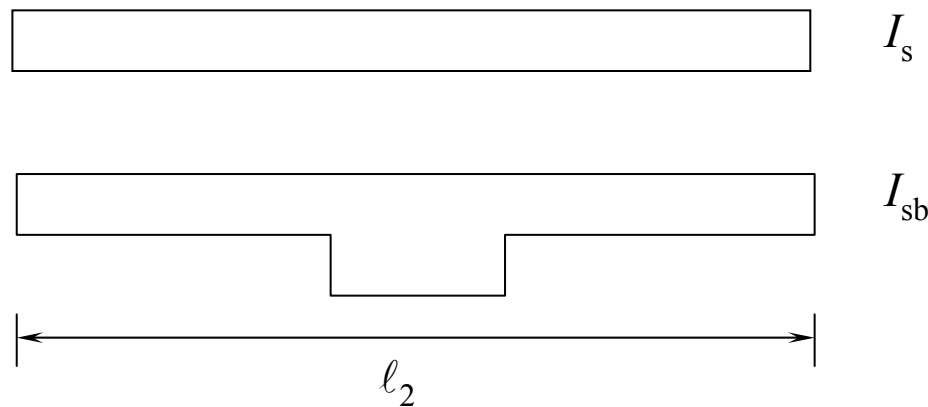

Fig. 12.39. Modification of Torsional Stiffness, K_t, Due to Beam.

$$K_{ta} = K_t \frac{I_{sb}}{I_s}$$

$$\theta_{EC} = \theta_C + \theta_{av}$$

where θ_{EC} = *rotation of equivalent column*

θ_C = *rotation of actual column*

θ_{av} = *average rotation of slab edge*

also, let,

K_{EC} = *stiffness of equivalent column*

$\sum K_C$ = *sum of actual stiffness of columns above and below the slab*

K_T = *sum of torsional stiffness of the perpendicular torsion members*

From the basic stiffness equation, we have,

$$K = M / \theta \quad \Rightarrow \quad \theta = M / K$$

The above equation for θ_{EC} may be written as follows:

$$\frac{M}{K_{EC}} = \frac{M}{\sum K_C} + \frac{M}{K_T}$$

$$\frac{1}{K_{EC}} = \frac{1}{\sum K_C} + \frac{1}{K_T}$$

If no torsion member is considered, $1 / K_T$ becomes infinity and K_{EC} is reduced to zero (or near a roller support).

Pattern Loading

According to ACI 13.7.6, the analysis may be simplified under the following conditions by avoiding pattern loading:

- If the live load is not exceeding three-fourth of the dead load, no pattern loading is required and the analysis is carried out for full load on all the panels.

- In cases where pattern loading is required, three-fourth of the actual live load is used for analysis. However, the factored moments calculated in this way should not be less than those occurring with full factored live load on all panels.

Correction Of Moments To Face Of Supports

- For design, the negative moments must be evaluated at the critical sections defined in ACI 13.7.7.

- These critical sections in case of columns, drop panels and column capitals are shown in Fig. 12.40.

Critical sections for M⁻

$0.175\ell_1$

$c_1 > 0.35\ell_1$

ℓ_1

$\alpha/2$

α

(b)

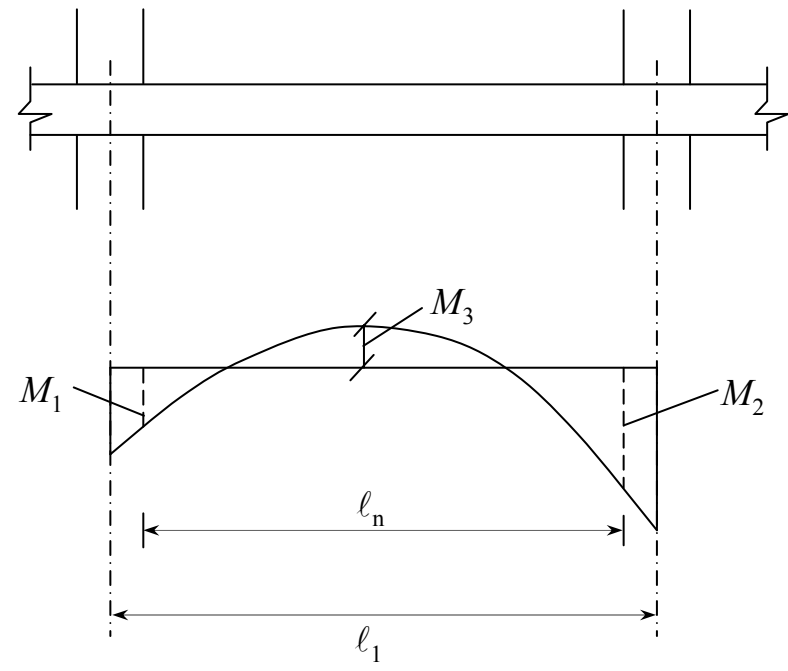M_3

M_1

M_2

ℓ_n

ℓ_1
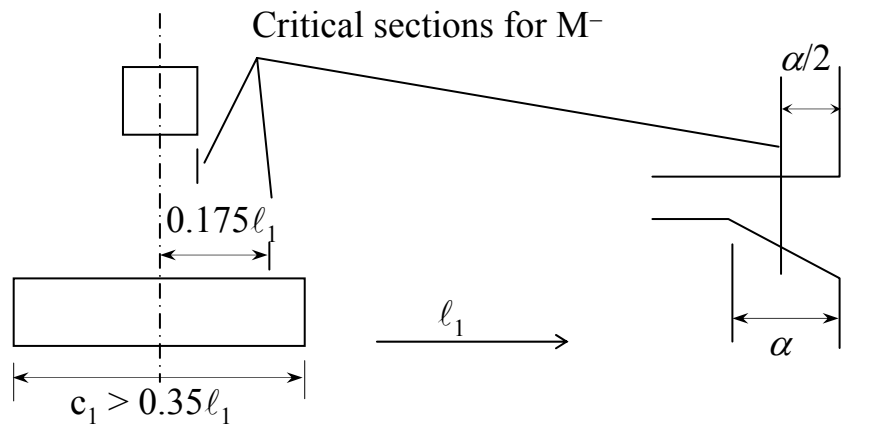
Fig. 12.40. Critical Sections For Negative Bending Moments.

If the slab system to be designed meets the requirements of the Direct Design Method, the total design moments in a panel can be reduced so that the absolute sum of the M^+ and the average M^- does not exceed the statical moment, M_o.

Referring to Fig. 12.40(b), the computed moments M_1, M_2 and M_3 are multiplied by 'F' where:

$$F = \frac{w_u \ell_2 \ell_n^2}{8} \bigg/ \left(\left| \frac{M_1 + M_2}{2} \right| + |M_3| \right) <= 1.0$$

Example 12.5

Find the properties of frame taken from a slab system and shown in Fig. 12.41 required for design of an interior design panel, having frames 8.5m on centers and columns of size 450mm × 250mm.

Service live load is to be 200 kg/m² for rooms and ordinary balconies, while it is to be considered 300 kg/m² for public balconies.

Center-to-center story height is 2.75m. Separate floor finish and permanent partition wall load is 150 kg/m².
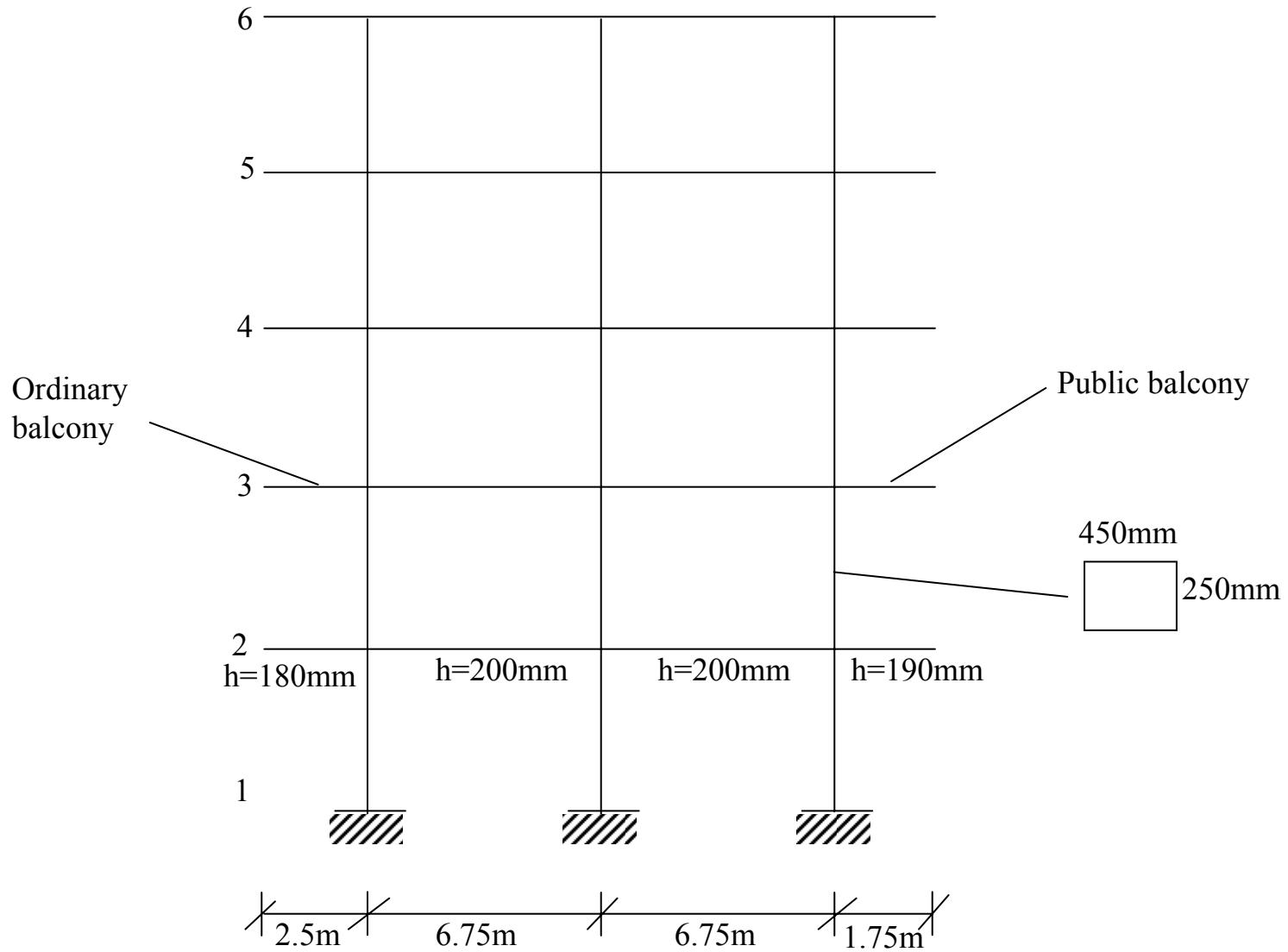
Fig. 12.41. Frame Dimensions For Example 12.5.

<u>Solution:</u>

Slab Loads:

200mm slab: q_D $\quad = 200/1000\,(2400) + 150 = 630\ \text{kg/m}^2$
$\quad\quad\quad\quad q_u$ $\quad = [1.2(630) + 1.6(200)] \times 9.81\,/\,1000 = 10.55\ \text{kN/m}^2$

190mm slab: q_D $\quad = 190/1000\,(2400) + 150 = 606\ \text{kg/m}^2$
$\quad\quad\quad\quad q_u$ $\quad = [1.2(606) + 1.6(300)] \times 9.81\,/\,1000 = 11.83\ \text{kN/m}^2$

180mm slab: q_D $\quad = 180/1000\,(2400) + 150 = 582\ \text{kg/m}^2$
$\quad\quad\quad\quad q_u$ $\quad = [1.2(582) + 1.6(200)] \times 9.81\,/\,1000 = 9.98\ \text{kN/m}^2$

For left balconies:
$\quad$ L.L. / D.L. $\;= 200\,/\,583 = 0.345 \quad < \quad 0.75$

For right balconies:
$\quad$ L.L. / D.L. $\;= 300\,/\,606 = 0.495 \quad < \quad 0.75$

∴   Pattern loading is not required to be considered.

9.98×8.5
=84.8kN/m

10.55×8.5
=89.7kN/m

89.7kN/m

11.83×8.5
=100.6kN/m

2.75m

2.75m

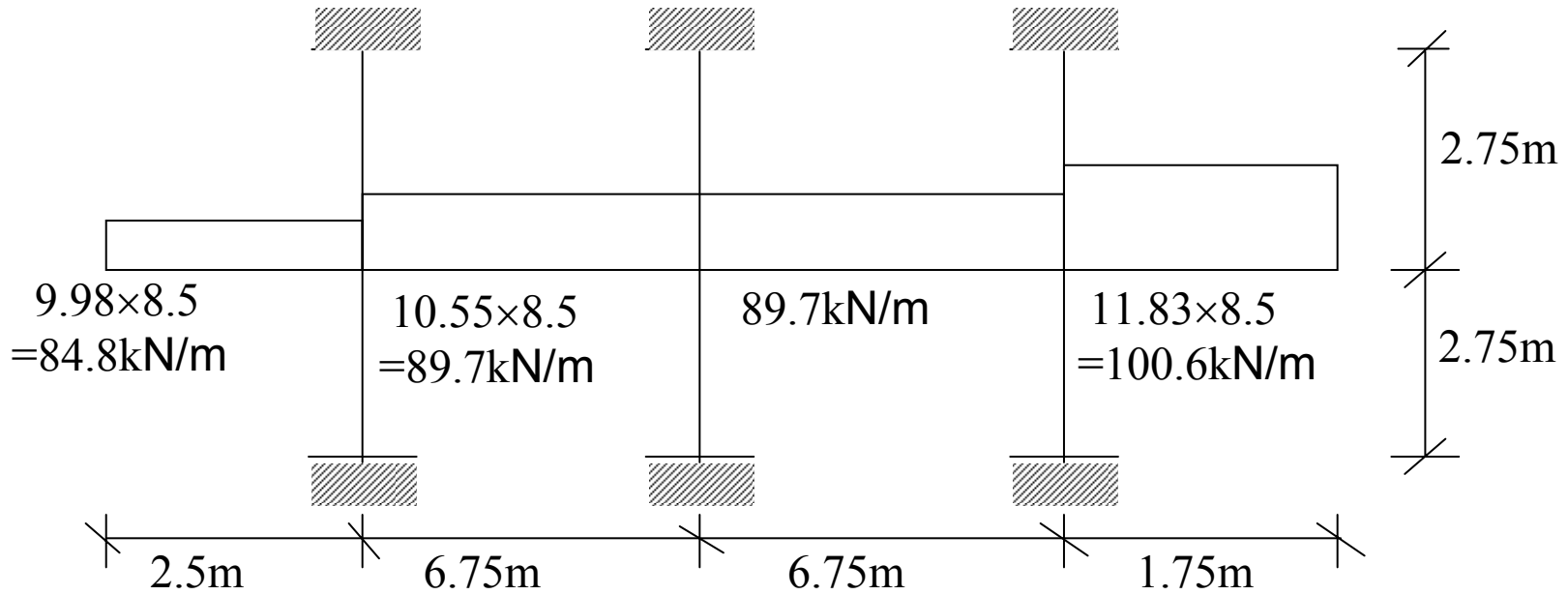2.5m   6.75m   6.75m   1.75m

Fig. 12.42. Isolated 2-D Frame For Analysis.

The frame may be simplified for analysis by replacing the cantilevers with the corresponding determinate moments, as shown in Fig. 12.43.
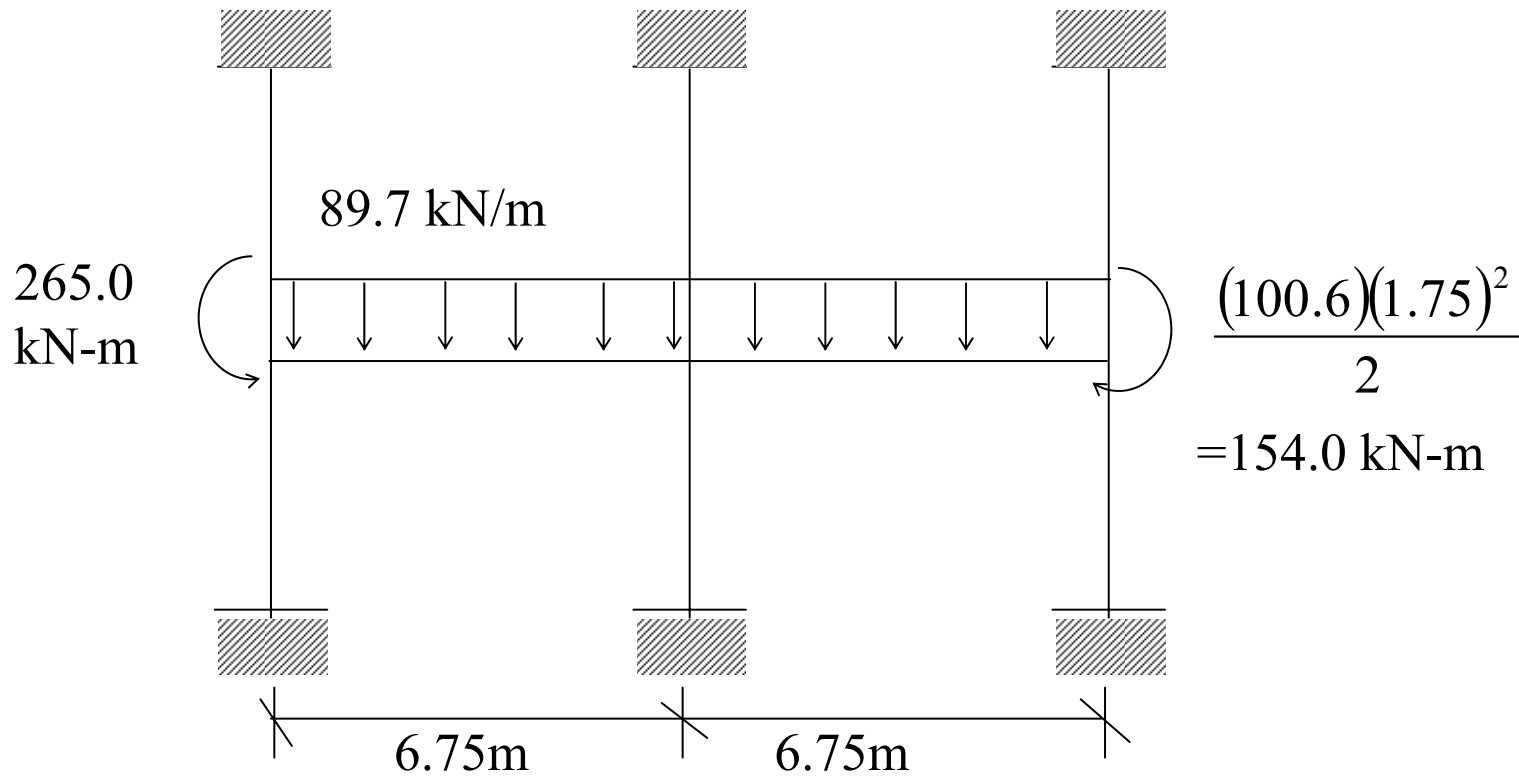
Fig. 12.43. Simplified Analysis Model.

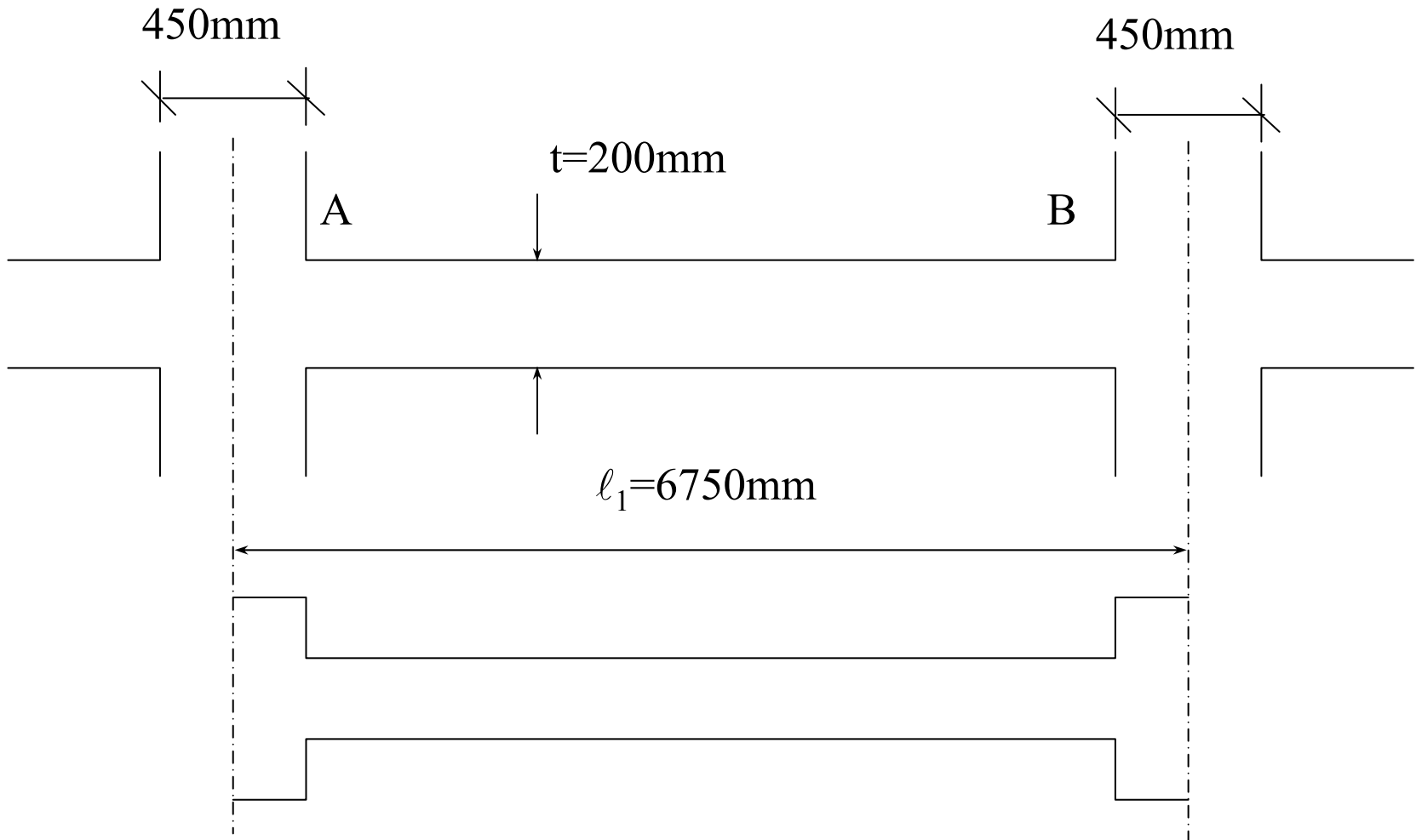The stiffness parameters for the slab-beam of Fig. 12.44 may be calculated as under:

450mm

450mm

t=200mm

A

B

ℓ_1=6750mm

Fig. 12.44. Moment of Inertia Diagram For Slab-Beam.

$$\frac{c_{1A}}{\ell_1} = \frac{c_{1B}}{\ell_1} = \frac{450}{6750} = 0.067$$

$$\frac{c_2}{\ell_2} = \frac{250}{8500} = 0.029$$

FEM (uniform load, q) $= m \times q \, \ell_{2w} \, \ell_1^2$

K_{sb} (stiffness) $= k \times EI_{sb} / \ell_1$ $t =$ slab depth

$$= \frac{k \, E \ell_2 t^3}{12 \ell_1}$$

Carryover factor $=$ C.O.F.

$$\alpha = \frac{\text{depth at drop panel}}{\text{depth of slab}}$$

$= 1.0$ for slab without drop panels

$$m = 0.09 \left(\frac{c_1}{\ell_1} \times \frac{c_2}{\ell_2} \right)^{0.015} \alpha^{0.24} \geq 0.083$$

$$= 0.09 \left(0.067 \times 0.029 \right)^{0.015} \times 1.0^{0.24} = 0.083$$

$$k = 5.3 \left(\frac{c_1}{\ell_1} \times \frac{c_2}{\ell_2} \right)^{0.05} \alpha^{0.9} \geq 4.0$$

$$= 5.3 \left(0.067 \times 0.029 \right)^{0.05} \times 1.0^{0.9} = 4.0$$

$$COF = 0.57 \left(0.067 \times 0.029 \right)^{0.02} \times 1.0^{0.37} = 0.503$$

$$\text{FEM} = m \times q \, \ell_{2w} \, \ell_1^2$$

$$= (0.083)(89.7)(6.75)^2 = 339.2 \text{ kN-m}$$

$$K_{\text{slab}} = k\,\frac{E\,\ell_{2w}\,t^3}{12\ell_1}$$

$$= \frac{4.0\,(8500)(200)^3}{6750 \times 1000}\,E = 3358\,E\ \text{kN-mm/rad}$$

The torsion member may be considered equal to a width of slab equal to the column dimension as shown in Fig. 12.45 and the relevant parameters are calculated as under:
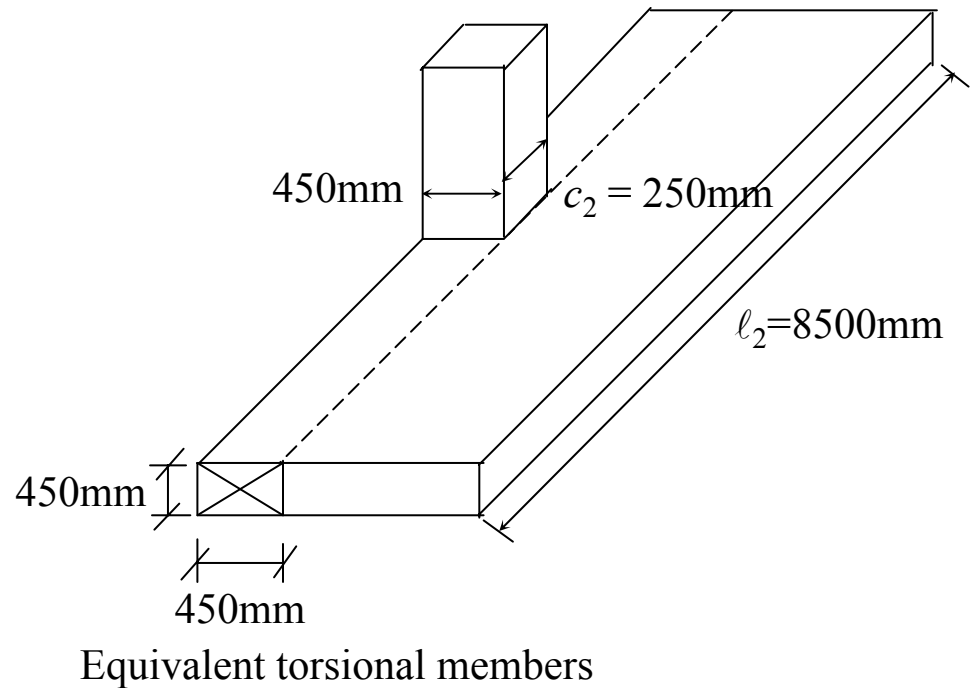

Equivalent torsional members

Fig. 12.45. Torsion Member For Example 12.5.

$$C = \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3}$$

$$= \left[1 - 0.63\left(\frac{200}{450}\right)\right]\frac{(200)^3(450)}{3} = 86400 \times 10^4 \text{ mm}^4$$

$$K_t = \sum\frac{9\,E_{cs}\,C}{\ell_2\left(1 - \dfrac{c_2}{\ell_2}\right)^3}$$

$$= (2)\frac{9\,E_{cs}\left(86400\times 10^4\right)}{(8500)\left[1 - \dfrac{250}{8500}\right]^3} \times \frac{1}{1000} = 2001\,E \quad \text{kN-mm/rad}$$

The properties of equivalent prismatic column may be found by using Fig. 12.46.

$$\ell_c \quad = \quad 2750 \text{ mm}$$

$$\ell_u \quad = \quad \ell_c - t_a - t_b$$
$$\quad = \quad 2550 \text{ mm}$$

$$t_a \quad = \quad 100 \text{ mm}$$

$$t_b \quad = \quad 100 \text{ mm}$$

$$\ell_c / \ell_u = 2750 / 2550 \quad = \quad 1.078$$

$$k_a \quad = \quad 4.0 \left(\frac{t_a}{t_b} \right)^{0.08} \left(\frac{\ell_c}{\ell_u} \right)^{2.7} \geq 4.0$$

for $t_a / t_b = 0.4$ to 2.2 and ℓ_c / ℓ_u up to 1.2

$$\quad = \quad 4.0 \, (1.0)^{0.08} \, (1.078)^{2.7} \quad = \quad 4.899$$

$$K_c \quad = \; k \times EI_c \,/\, \ell_c$$

$$= \; \frac{4.899\,(250)(450)^3}{12 \times 2750 \times 1000}\; E \;=\; 3382\,E \;\; \text{kN-mm/rad}$$

$$COF \;=\; 0.5\;\; \frac{\left(\ell_c \,/\, \ell_u\right)}{\left(t_a \,/\, t_b\right)^{0.08}} \;\; \geq \; 0.5$$

$$= \; 0.5\,\frac{1.078}{(1.0)^{0.08}} \;=\; 0.539$$

$$\frac{1}{K_{EC}} \;=\; \frac{1}{\sum K_C} \;+\; \frac{1}{K_T}$$

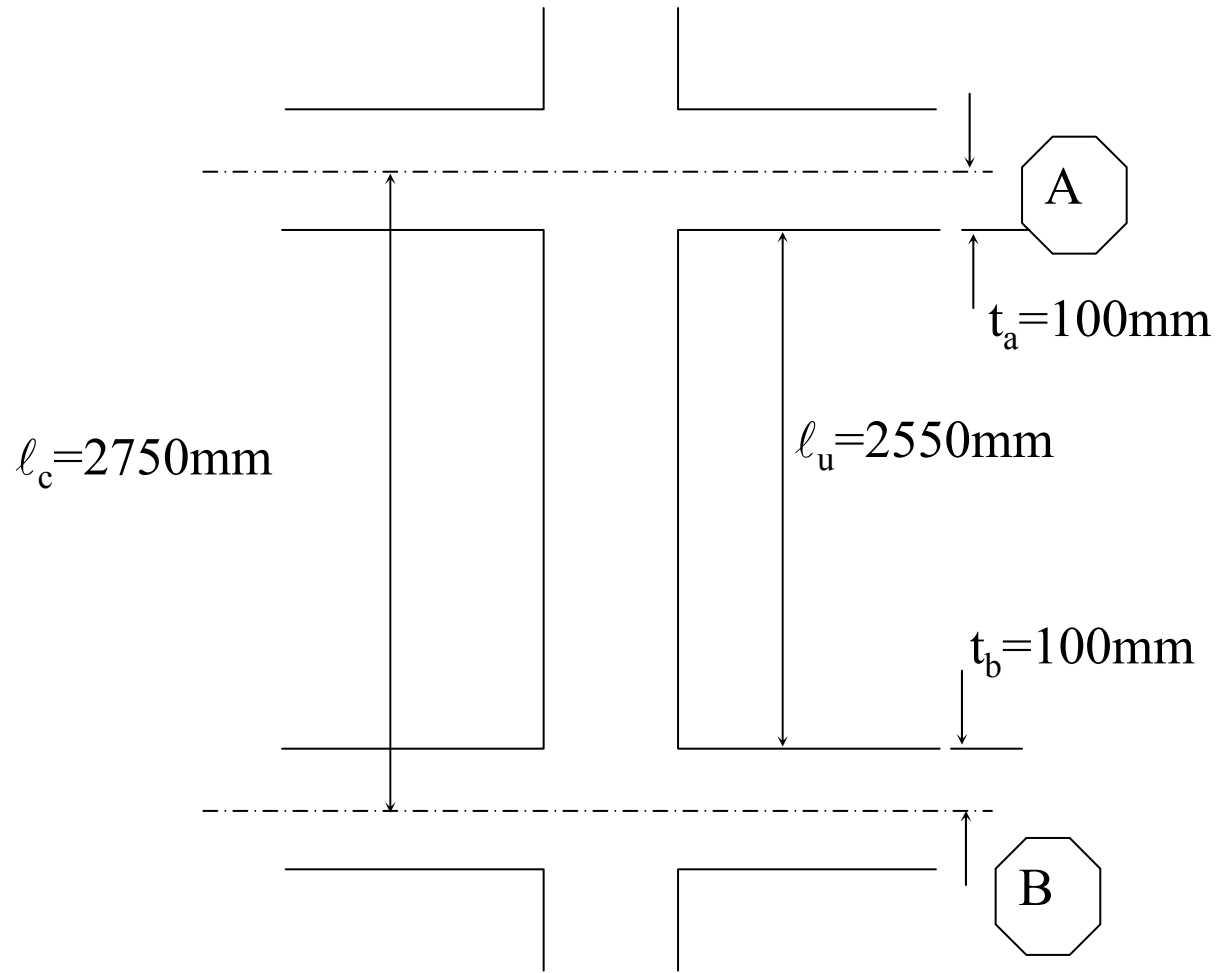$$= \; \frac{1}{2(3382\,E)} \;+\; \frac{1}{2001\,E}$$

Fig. 12.46. Column Properties For Example 12.5.

$$K_{ec} \quad = \quad 1545\,E$$

Considering the frame of Fig. 12.47, the distribution factors are calculated as under:

$$DF_{BD} = \quad DF_{FD} = \quad \frac{215\,E}{102\,E + 215\,E} \quad = \quad 0.68$$

$$DF_{DB} = \quad DF_{DF} = \frac{215\,E}{215\,E + 215\,E + 102\,E} = \quad 0.404$$

After analysis of frame of Fig. 12.47, perform the following steps and design the system:

- Correct moments to face of the supports.
- Distribute into column strips and middle strips by using the expression of the direct design method.
- Design the column and middle strip slabs.
- Carry out detailing of the reinforcement.
- The column moments may be evaluated by satisfying the equilibrium at their junction with the slab. The unbalanced slab moments are distributed into the top and bottom columns according to ratio of their stiffness.

339.2 kN-m (F.E.M.)

265.0 kN-m (App. Moment)

339.2 kN-m (F.E.M.)

154.0 kN-m (App. Moment)
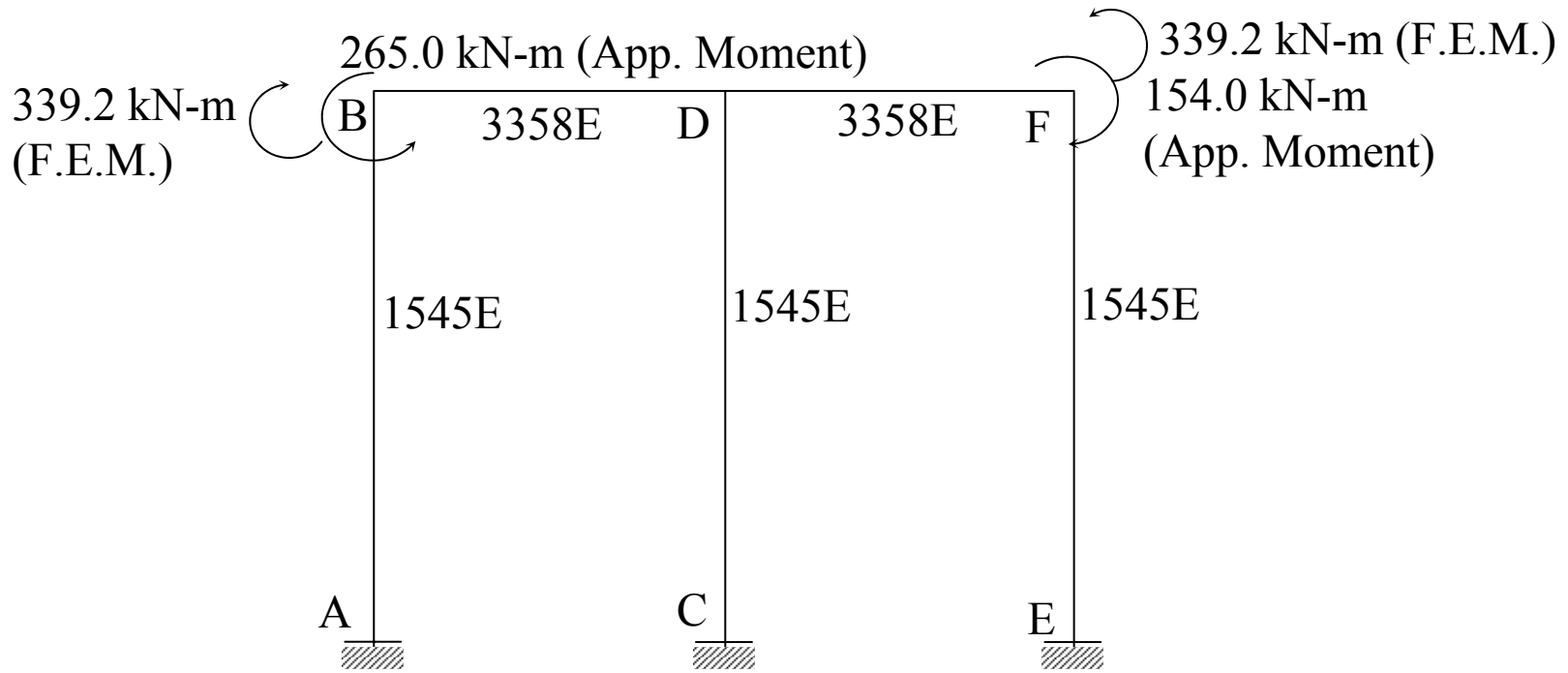
B   3358E   D   3358E   F

1545E   1545E   1545E

A   C   E

Fig. 12.47. Final Analysis Frame For Example 12.5.