# Steel Structures

## M.Sc. Structural Engineering

## SE-505

Lecture # 01

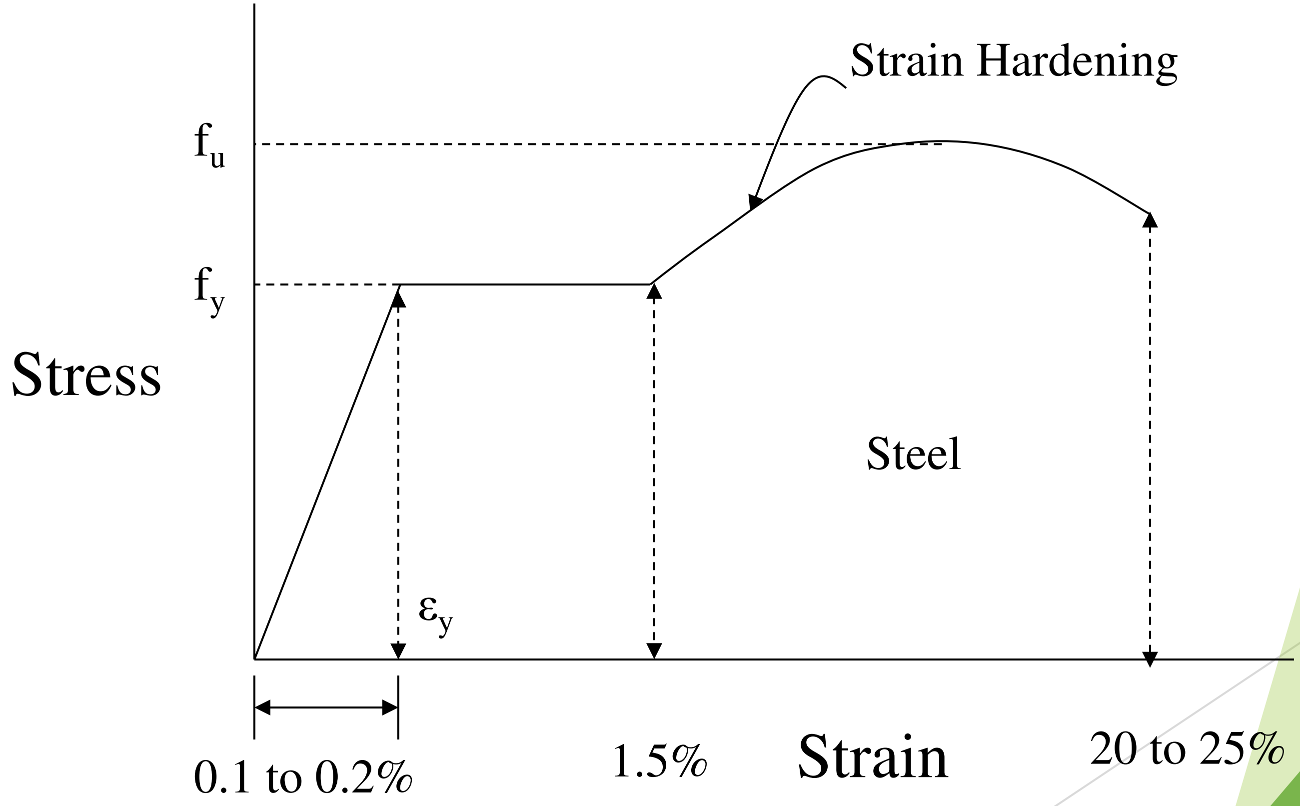
10th September 2018

Plastic Analysis and Design of Structures

**Dr. Qasim Shaukat Khan**

# Steel Structures

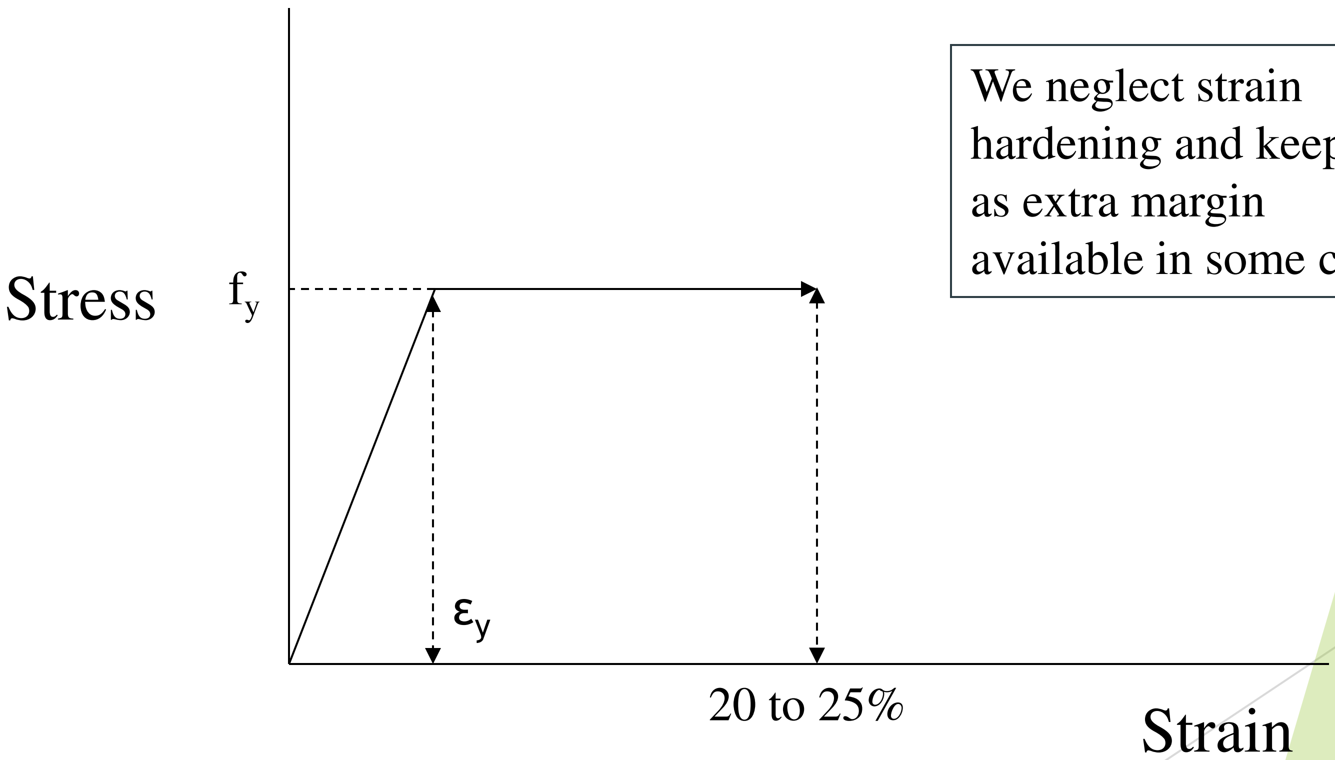
Inelastic or Ductile Design of Structures
(Plastic Analysis and Design)




Typical Stress Strain Diagram

# Steel Structures

## Inelastic or Ductile Design of Structures



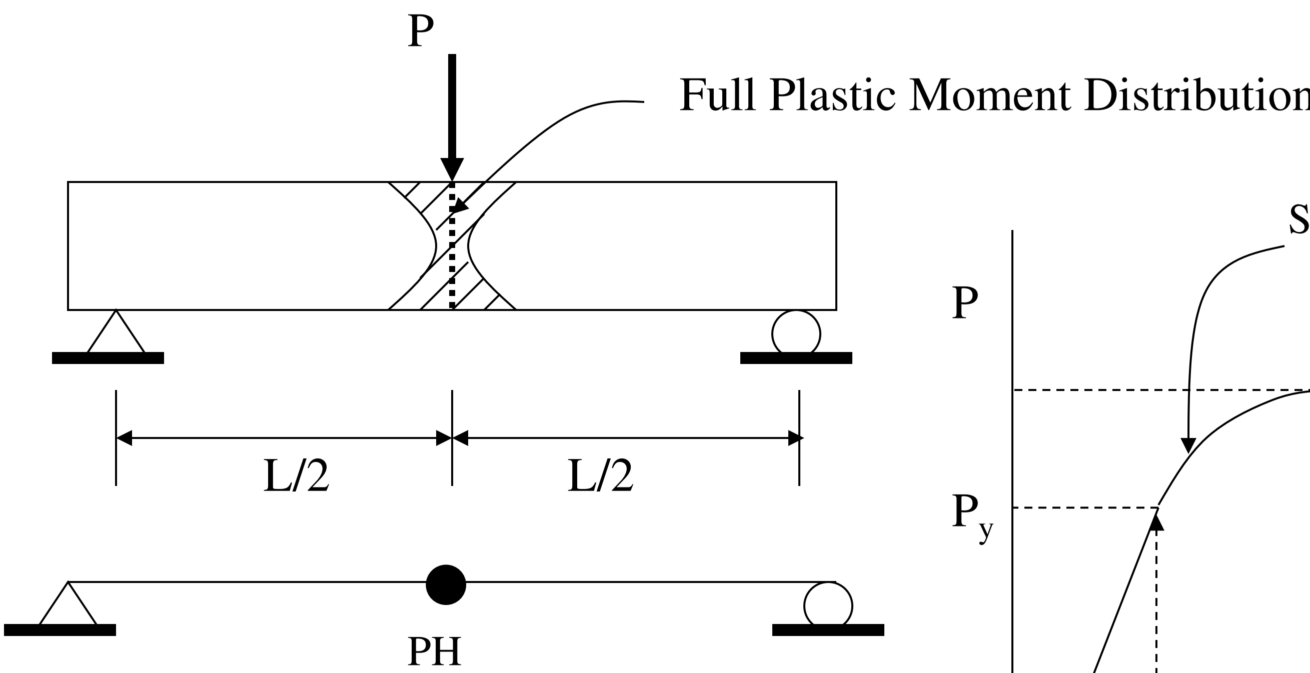

We neglect strain hardening and keep it as extra margin available in some cases

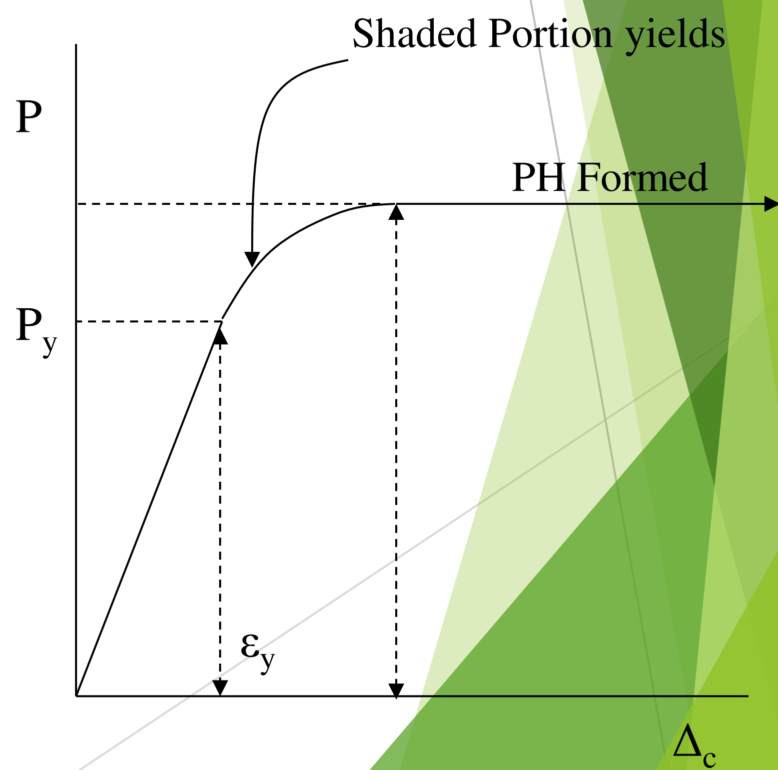
Assumed Bilinear Stress Strain Diagram

# Steel Structures



## Load Deflection Curve for flexure




Rotation becomes free at the section of full plastic moment (Mp).

$P_y$ = Load at which only outer fiber yields

- **Elastic range:** In case of beams, when strain is more than εy, fibers on the outer side will be in inelastic range and inner fibers will be in elastic range.
- **Strain at extreme fibers at max. moment section is greater than εy but yielding does not penetrate full depth:** Outer fibers are in inelastic range do not provide resistance to deformation. Inner fibers are in elastic range and provide resistance to deformation.
- **Yielding penetrated to full depth at max. moment section:** Yielding has penetrated full depth, strain in beam is increasing significantly and rotation has become free, at this point plastic hinge will form. Beam will become internally unstable but will not fail.
- Beam will start settling down, once rotation capacity is exhausted, beam will fail.

# Steel Structures

## Load deflection curve (contd…)

When load is less, stresses are lesser than $F_y$, load and deflection are linearly related. For the simply supported beam with a point load in center:

$$\Delta = \frac{PL^3}{48EI} \qquad \Delta \propto P \qquad F_y = \frac{My}{I} \qquad F_y = \frac{M_y}{S}$$

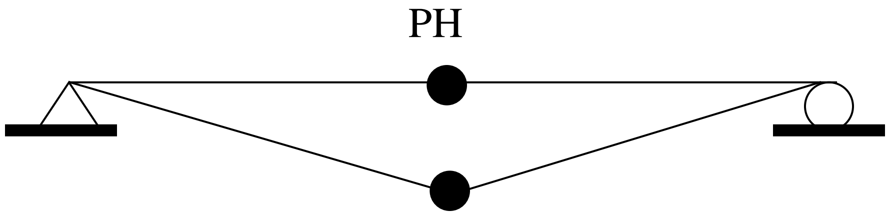
When $P > P_y$, yielding will penetrate inside at the maximum moment section.

The moment of inertia of the elastic portion reduces and hence deflections start increasing at a greater rate.

# Steel Structures

## Full Plastic Moment Capacity, $M_P$

At the section of $M_P$ moment becomes constant and rotation becomes free. Section can rotate without further increase of load. This particular section is called **“Plastic Hinge”.**




Mechanism

After mechanism, deformation will become very large at constant load until rotation capacity of section is exhausted and after this final failure will take place.

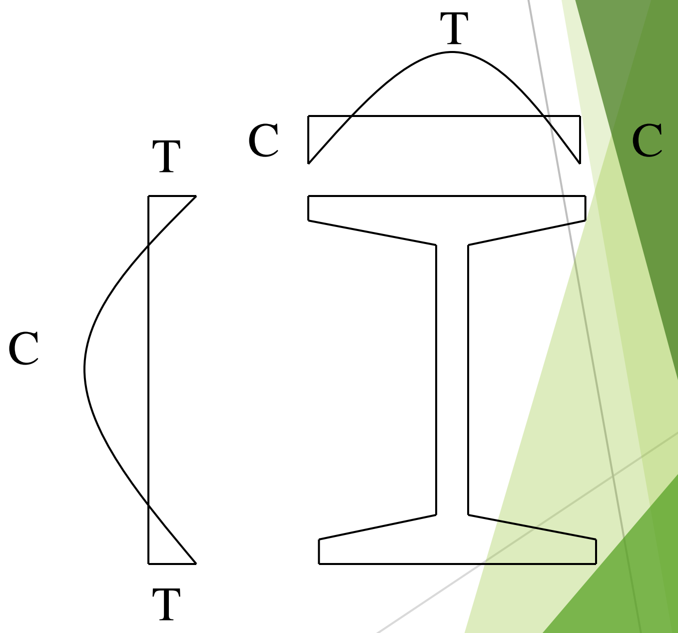
# Steel Structures

## Residual Stresses

Hot rolled sections have residual stresses due to differential cooling during manufacture.

Parts smaller in thickness will cool first. Thin sections (web) will cool quickly developing compressive stresses. Thick sections (intersection of web and flange) will cool slowly developing tensile stresses.

Maximum residual stress in hot rolled section is upto 30-40% of $F_y$.



# Steel Structures

## Bending Theory

<u>Assumptions</u>

1. Material is homogeneous and isotropic.
2. Member is subjected to bending moment only.
3. The ordinary bending formula is developed for objects symmetrical at least about one axis.
4. Material obeys Hooke's law (In case of Plastic bending theory, Hooke's law is not applicable).
5. Plane section remains plane, even after bending

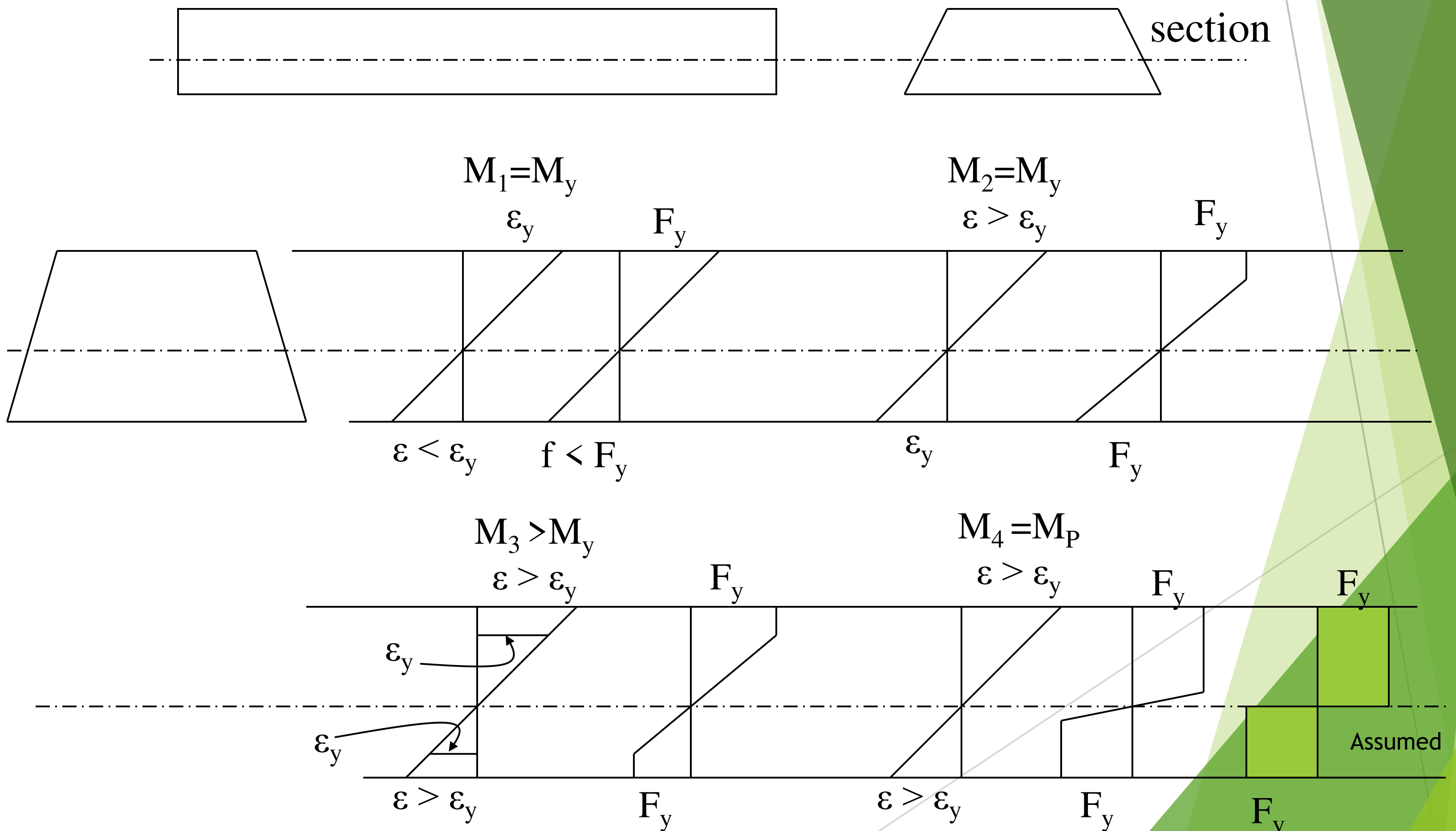
$$\varepsilon = \frac{y}{R} \Rightarrow \varepsilon \ \alpha \ y$$

y = distance of fiber where strain is to be calculated.
R = Radius of curvature

# Steel Structures



## Inelastic Bending (For Different Stages of Loading)

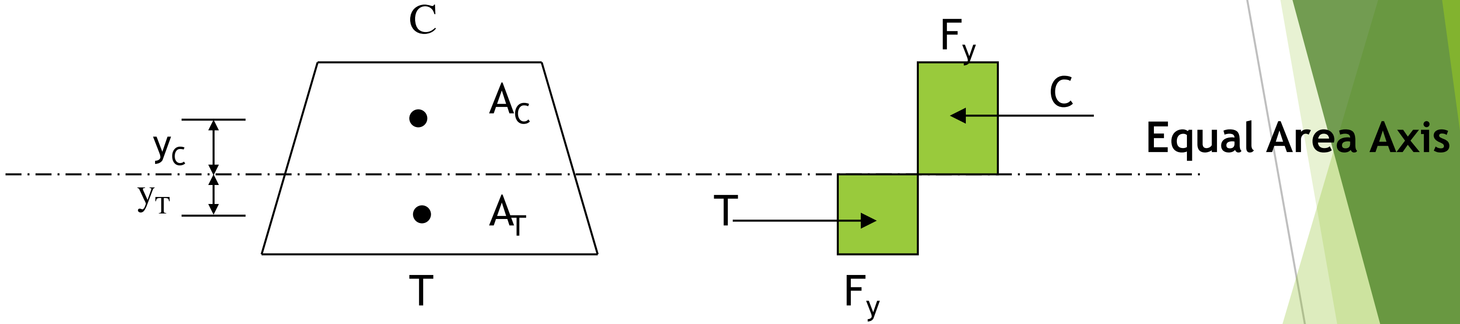


# Steel Structures



## Inelastic Bending (contd…)

When section is fully plastic, N.A. may not pass through centroid of cross section




Total tension = Total Compression

$$A_T \times F_y = A_C \times F_y$$

$$\frac{A}{2} = A_T = A_C$$

The axis dividing the area into two equal halves is called the equal area axis. The stress changes its sense at this axis.

# Steel Structures

## Inelastic Bending (contd…)

$$M_P = C \times y_c + T \times y_T$$

$$M_P = F_y \times \frac{A}{2} \times y_c + F_y \times \frac{A}{2} \times y_T$$

$$M_P = F_y \times \frac{A}{2} \times \left(y_c + y_T\right)$$

First moment of area about equal area axis, called **Plastic Section Modulus, Z**

$$= \frac{A}{2} \times \left(y_c + y_T\right)$$

# Steel Structures

## Inelastic Bending (contd…)

$$\boxed{M_P = F_y \times Z}$$

$$M_y = F_y \times S$$

$M_y$ = Moment at which yielding starts at the outer edge

# Steel Structures

## Shape Factor (F)

$$F = \frac{Z}{S} = \frac{M_P}{M_y} > 1.0$$

Shape factor depends upon shape of cross section.

Rectangular section, F = 1.5 $M_P = 1.5\ M_y$

Circular section, F = 1.698

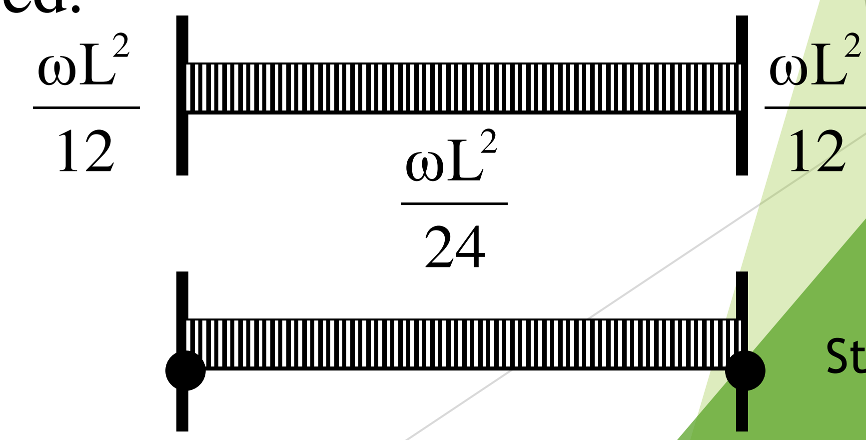
Diamond section, F = 2

W-Section, F = 1.1 to 1.18 **(1.15)**

# Steel Structures

## Advantages of Plastic Analysis and Design

1. Difference between load analysis and strength evaluation is removed (unlike LRFD). Analysis and design both are carried out in inelastic range.
2. Reserve strength of most heavily stressed section and other less stressed sections in case of indeterminate structures is utilized.




Still Stable

> Redistribution of moments is only there in plastic analysis

# Steel Structures

Advantages of Plastic Analysis and Design (contd…)

3. True collapse mechanism can be accurately predicted.
4. For smaller structures plastic method has lesser calculations.
5. Overall F.O.S is same as in other methods.
6. At service stage the structure is still elastic.
7. At working load, deflections will be less.

# Steel Structures

## Disadvantages of Plastic Analysis and Design

1. Gives upper bound solution. Can give unsafe design if all failure possibilities are not considered.

2. Principle of super position is not valid. So we have to make separate analysis for all the load combinations.

3. Deflections may be critical in some cases.

4. The local and lateral stability becomes much more important. Where PH is formed there is rotation to reach to other PHs. So chances of local and overall buckling are high. Every section and connection has a particular rotation capacity.

5. Can become very lengthy for large size frames.

# Steel Structures

## Ductility

**"Measure of deformation capacity of a member before final failure"**

Deformation is mostly in inelastic range. For flexural section ductility is usually measured in terms of rotation capacity.

## Compact Section

**"A section which does not show stability problem before reaching the plastic moment and still provides rotation capacity at a constant moment"**

For plastic analysis and design section must be compact because we need rotation capacity.

$$L_b \leq L_{Pd}$$

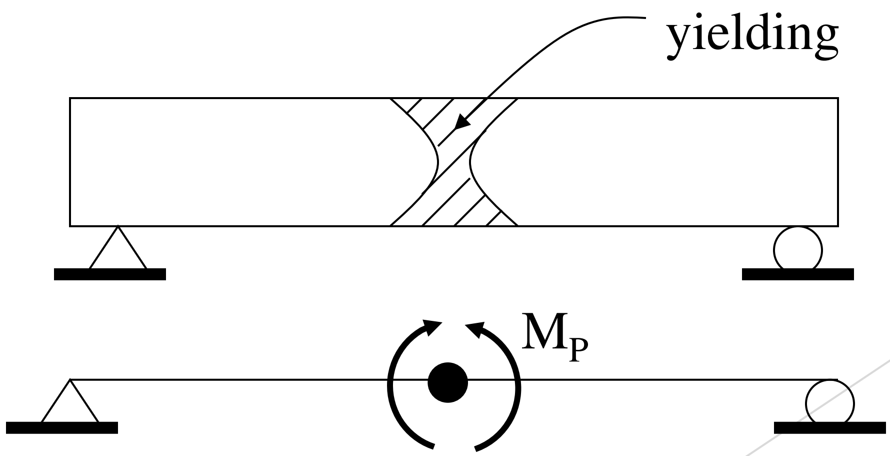
# Steel Structures

## Type of Steel

Mild steel and high strength steel can be used for plastic design.

## Plastic Hinge

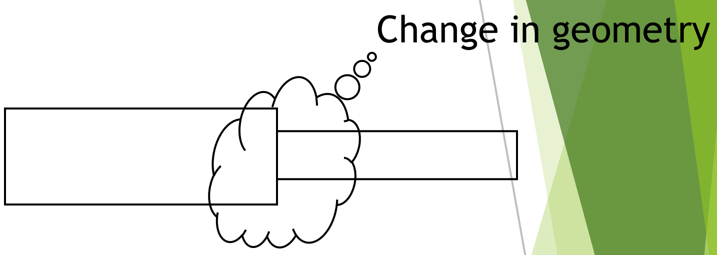
When a section of a structural member reaches a maximum value of moment (Full Plastic Moment) and free rotation can occur at this constant moment, we can say that a plastic hinge is developed at this section.



# Steel Structures

Possible Locations of Plastic Hinge

1. Under the point loads
2. At connection of members
3. At change of geometry
4. At the point where shear force changes sign.
   (in case of UDL)



# Steel Structures

## Length of Plastic Hinge

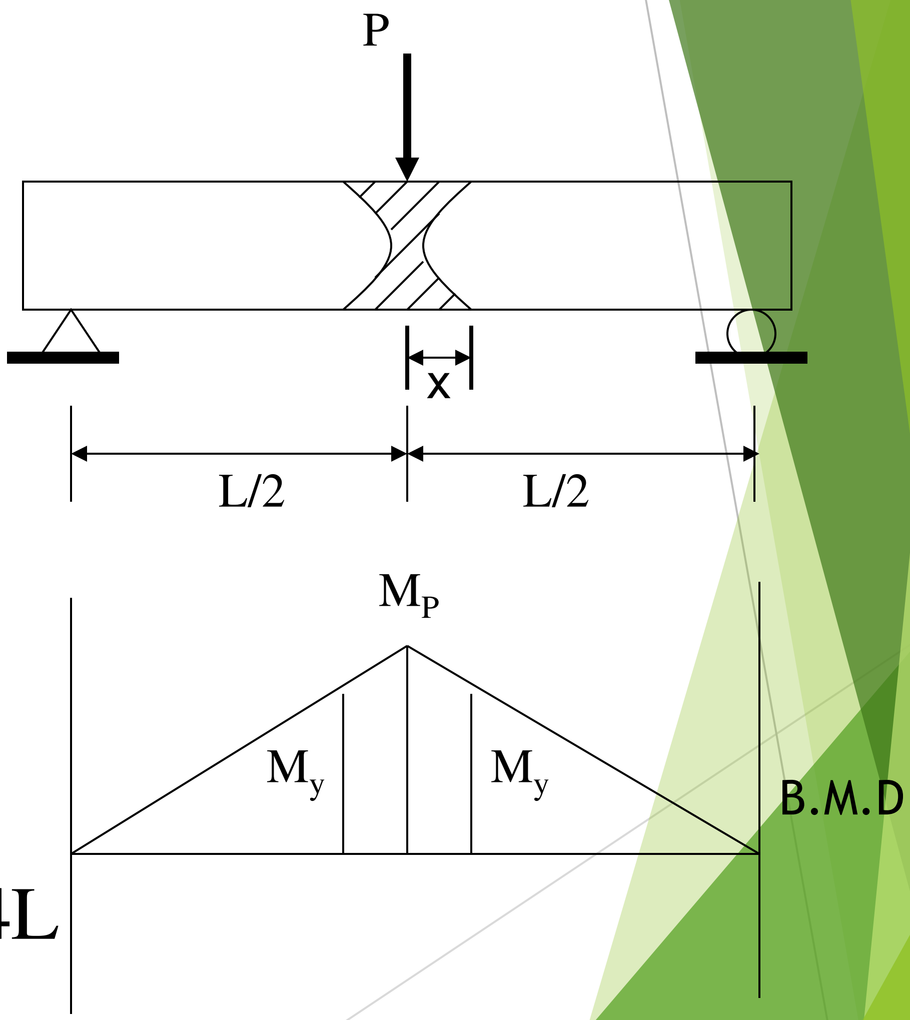
$$\frac{M_P}{M_y} = \frac{L/2}{L/2 - x}$$

$$\frac{M_P}{M_y} = F = 1.15$$ For W-section

$$1.15 = \frac{L/2}{L/2 - x}$$

$$x = 0.0652L$$

Length of PH = 2x $= 0.1304L$



# Steel Structures

## Length of Plastic Hinge

Length of PH = 2x $= 0.1304L$

$$2x = \frac{L}{7.6}$$

For rectangular section, F = 1.5

$$2x = \frac{L}{3}$$

# Concluded