Steel Structures

Plastic Design

Lec-5

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Steel Structures

Design of Frames

It consists of designing:

Beams, Columns, Connections, Columns bases, etc.

Connections:

"The connection which can transfer full moment from beam to column and vice versa along with shear is **Moment Connection**"

Requirements for Connection

1. A hinge can develop and can be maintained at the connection (there must be enough rotation capacity).
2. Sufficient strength against moment and shear.

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Moment Connections:

All the frame connections have to transfer 100% shear force from the beams to other members. The amount of moment to be transferred varies depending on the rotational restraint at the joints.

- Fully Restrained Moment Connections (Rigid or continuous frame connections transfer 100% moment along with 100% shear).

- Partially Restrained Moment Connections (Beam column connections under cyclic loads and earthquake loads transfer approx., 20-90% moment along with 100% shear).

- Shear Connections (Connections of beams, girders and trusses transfer less than 20% moment along with 100% shear).

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EXTRA RESTRICTION FOR INELASTIC ANALYSIS AND DESIGN

The extra provisions for the inelastic design are provided in AISC Appendix 1.

***Inelastic analysis* is not allowed for design according to the ASD provisions but moment redistribution is allowed.**

In LRFD, structure is analyzed within elastic range. However, inelastic behavior, ultimate failure modes and redistribution of forces after elastic analysis is considered. Plastic design is somewhat similar to LRFD but analysis is also performed considering collapse mechanism of the structure.

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1. MATERIALS

Members undergoing plastic hinging should have a ***specified minimum yield stress*** not exceeding 450 MPa.

2. MOMENT REDISTRIBUTION

Beams composed of *compact sections,* may be proportioned for nine-tenths of the −ve moments at points of support, produced by the *gravity loading* computed by an *elastic analysis,* provided that the max. +ve moment is increased by one-tenth of the average −ve moments.

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This reduction is not allowed for moments produced by loading on cantilevers and for design according to plastic methods.

If the negative moment is resisted by a *column* rigidly framed to the beam or girder, the one-tenth reduction may be used in proportioning the column for combined axial *force* and flexure, provided that the axial force does not exceed $0.15\phi_c F_y A_g$ for LRFD or $0.15 F_y A_g/\Omega_c$ for ASD.

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Cross-Section

Only doubly symmetrical I-shaped section is allowed at the plastic hinge locations. The elements of the cross-section must have width to thickness ratio (b/t_f) less than or equal to λ_{pd} which is generally equal to λ_p with some modifications as under;

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Flanges and webs of members subjected to plastic hinging in combined flexure and axial compression should be compact and must also satisfy the following requirements:

(a) The flanges of symmetric I-sections must satisfy the following usual condition of compactness:

$$\frac{b_f}{2t_f} \leq 0.38\sqrt{\frac{E}{F_y}} = 10.8 \text{ for A36steel}$$

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(b)For webs of doubly symmetric wide flange members and rectangular *HSS* in combined flexure and compression

(i) For negligible P_u

$$\frac{h}{t_w} \leq 3.76\sqrt{\frac{E}{F_y}} = 107 \quad \text{for A36steel}$$

(ii) For $P_u/\phi_c P_y \leq 0.125$

$$\frac{h}{t_w} \leq 3.76\sqrt{\frac{E}{F_y}}\left(1 - \frac{2.75P_u}{\phi_b P_y}\right)$$

Pu is the required axial strength in compression in N and ϕ_c is the resistance factor for compression =0.9 and Py is the yield strength in N

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(iii) For $P_u/\phi_b P_y > 0.125$

$$\frac{h}{t_w} \leq 1.12\sqrt{\frac{E}{F_y}}\left(2.33 - \frac{P_u}{\phi_b P_y}\right) \geq 1.49\sqrt{\frac{E}{F_y}}$$

4. STABILITY AND SECOND-ORDER EFFECTS

Continuous *beams* not subjected to axial *loads* and that do not contribute to lateral *stability* of framed structures may be designed based on a *first-order inelastic analysis* or a plastic *mechanism* analysis.

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Braced frames and ***moment frames*** may be designed based on a ***first-order inelastic analysis*** or a plastic *mechanism* analysis provided that ***stability*** and ***second-order effects*** are taken into account.

Structures may be designed on the basis of a second-order ***inelastic analysis***.

For *beam-columns, connections* and connected members, the *required strengths* shall be determined from a second-order inelastic analysis, where equilibrium is satisfied on the deformed geometry, taking into account the change in ***stiffness*** due to yielding.

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1. Braced Frames

In *braced frames*, the braces should be designed to remain elastic under the *design loads*.

The required axial strength for *columns* and compression braces must not exceed $\phi_c(0.85 F_y A_g)$, where, $\phi_c = 0.90$ (LRFD).

2. Moment Frames

In *moment frames*, the required axial strength of columns must not exceed $\phi_c(0.75 F_y A_g)$, where, $\phi_c = 0.90$ (LRFD).

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5. COLUMNS AND OTHER COMPRESSION MEMBERS

Design by inelastic analysis is allowed if the column slenderness ratio, *L/r*, does not exceed,

$$4.71\sqrt{E / F_y}$$

where,

L = laterally unbraced length of a member, mm

r = governing radius of gyration, mm

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6. BEAMS AND OTHER FLEXURAL MEMBERS

The required moment strength, M_u, of *beams* must not exceed the *design strength*, ϕM_n, where

$$M_n = M_p = F_y Z < 1.6 F_y S \qquad ; \; \phi = 0.90 \text{ (LRFD)}$$

Design by inelastic analysis is only permitted for compact sections.

The *unbraced length*, L_b, of the flexure member is defined as the length between points braced against lateral displacement of the compression flange or twist of the cross-section.

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Unbraced length for members loaded simultaneously with axial compression and flexure is defined as the length between the points braced against lateral displacement in the minor axis and twist of the x-section. For members containing *plastic hinge* *Lb* shall not exceed L_{pd} ,determined as follows.

$$L_{pd} = \left[0.12 + 0.076\left(\frac{M_1}{M_2}\right)\right]\left(\frac{E}{F_y}\right) r_y$$

where

M_1 = smaller moment at end of unbraced length of beam, N-mm

M_2 = larger moment at end of unbraced length of beam, N-mm

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r_y= radius of gyration about minor axis, mm

(M_1/M_2) is positive when moments cause *reverse curvature* and negative for *single curvature*.

7. MEMBERS UNDER COMBINED FORCES

Inelastic analysis is not allowed for members subject to torsion and combined torsion, flexure, shear and/or axial force.

8. CONNECTIONS

Connections adjacent to plastic hinging regions must be designed with sufficient strength and ductility to sustain the *forces* and deformations imposed under the required *loads*.

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Effective Length Factor, K:

Ratio of effective length to the unsupported length. This depends on end conditions of column and whether side sway is permitted or not.

Greater the K-value, greater is the effective length and slenderness ratio and hence smaller is the buckling load.

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Moment magnification factors (B1 and B2) are used to empirically estimate the magnification produced in the column due to 2nd order effects.

No-Sway Moment Magnification

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}} \geq 1.0$$

P # 159

P_r = required second order axial strength

$\approx P_{nt} + P_{lt}$ (based on first order estimate)

Sway Moment Magnification For Plastic Design

$$B_2 = 1 + 2.5 \frac{\alpha (\sum P_{nt})(\sum H) L \theta}{(\sum P_{e2})(\sum M_p \phi)}$$

Not given in code

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Design

Sway Moment Magnification

θ = Rotation of columns for critical collapse mechanism.

ϕ = Rotation for all hinges, for all beams & columns, for the critical collapse mechanism above column under consideration.

$\sum P_{nt}$ = Total storey vertical load

$\sum H$ = Total horizontal load on story

$\sum P_{e2}$ = Sum of critical buckling loads of all columns of the storey considering K unbraced.

L = Height of Column (storey height)

α = 1.00 for LRFD and Plastic Design

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Design

Required Plastic Section Modulus Revised For 2nd Order Effects

$$Z = Z_t\left(\frac{P_u}{\phi_c P_n} + 0.889\right) \quad \text{for} \quad \frac{P_u}{\phi_c P_n} \geq 0.2$$

$$Z = Z_t\left(\frac{P_u}{\phi_c P_n} + 0.889B\right)$$

This includes moment magnification. If we use this expression trials will be reduced.

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Design
Required Plastic Section Modulus Revised For 2nd Order Effects

$$Z = Z_t\left(0.5\frac{P_u}{\phi_c P_n} + 1.0\right) \quad \text{for} \quad \frac{P_u}{\phi_c P_n} < 0.2$$

$$Z = Z_t\left(0.5\frac{P_u}{\phi_c P_n} + 1.0B\right)$$

This includes moment magnification. If we use this expression trials will be reduced.

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Continuous Beam Design

1. Draw structure with factored loads
2. Draw BMD for primary structure
3. Establish failure mechanism
4. Draw –ve BMD
5. Calculate reactions
6. Find Z_{req} and $d_{\text{min}} = L/22$ for S. S. beams

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Continuous Beam Design

7. Select section with values of point # 6

8. Check for local and overall stability

9. Check for $V_{u\ max}$

10. Decide bracing

11. Decide splicing.

12. Calculate deflection at service loads.

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Types of Splicing

1. Shear Splicing
   “Which transfers only shear”. Only web is connected. Shear splicing can be provided at zero moment region i.e. point of contraflexure

2. Moment splicing
   “Which transfers both shear and moment”. Web and flanges both are connected.

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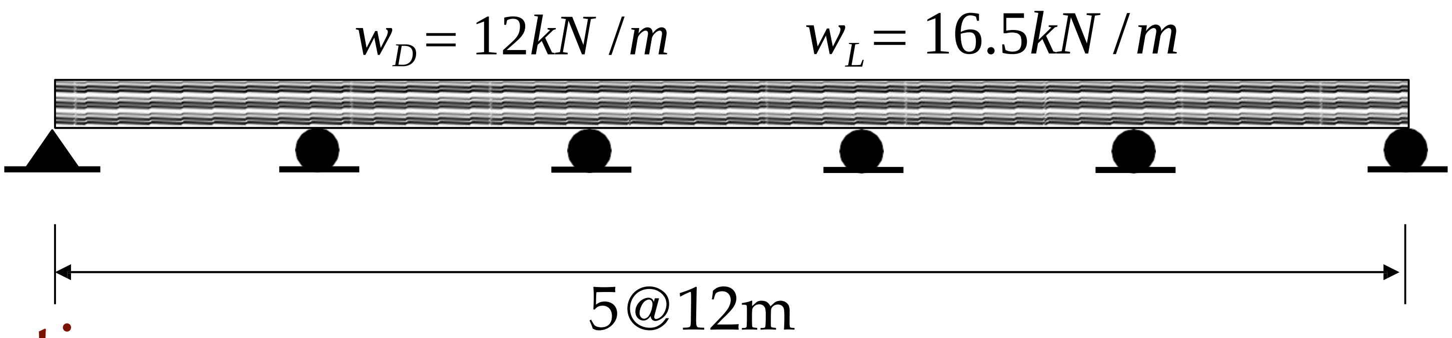
Example: Design the continuous beam of uniform section. Top flange is continuously connected with slab.



Solution:

$$w_u = 1.2 \times 12 + 1.6 \times 16.5 = 40.8kN/m$$

$$M_s = \frac{wL^2}{8} = \frac{40.8 \times 12^2}{8} = 734.4kN-m$$

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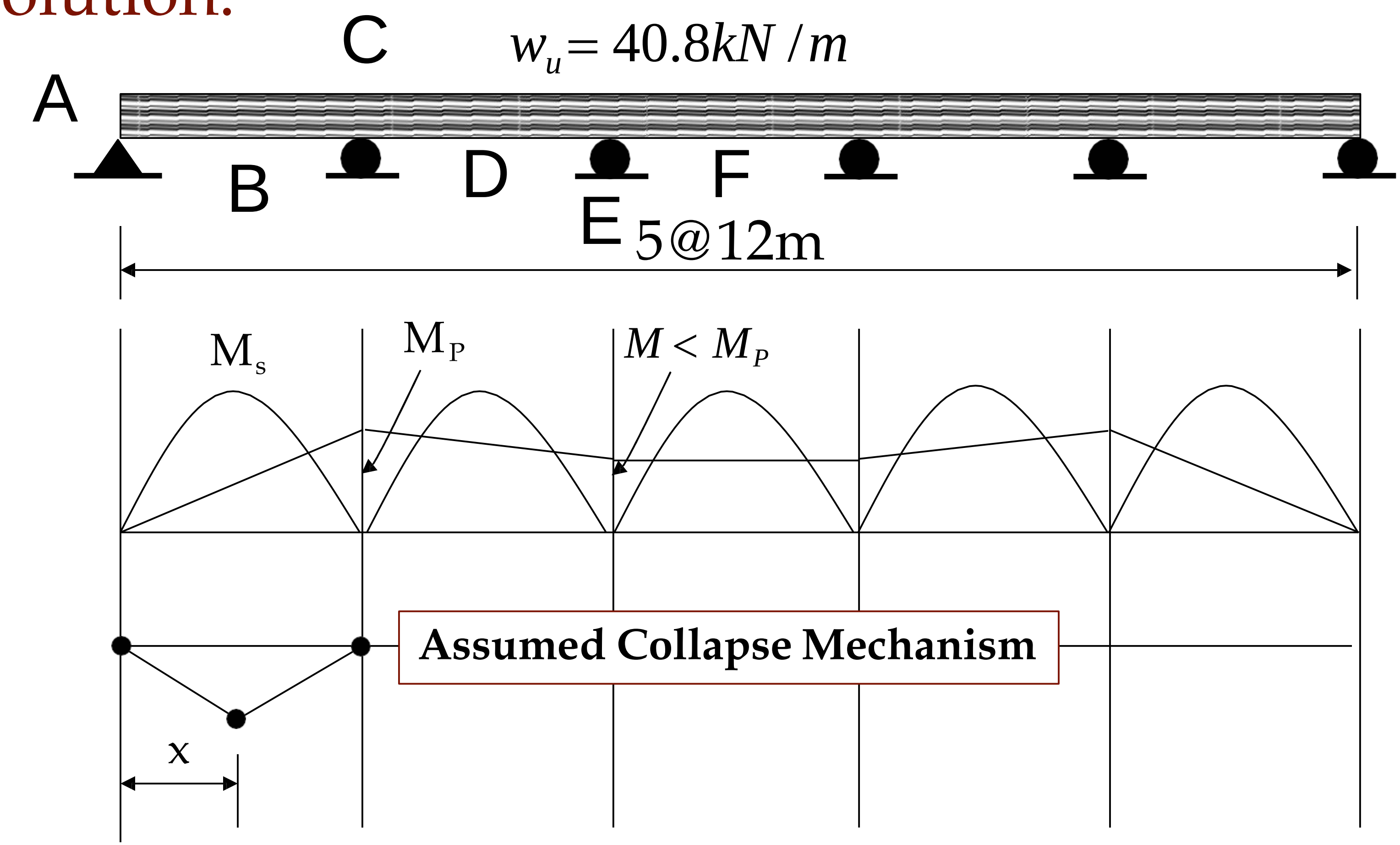
Solution:



Mechanism is approximately same as propped cantilever beam

$x = 0.414L = 4.97m$ $\qquad M_P = 0.686M_s = 503.8kN-m$

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Solution: (contd…)

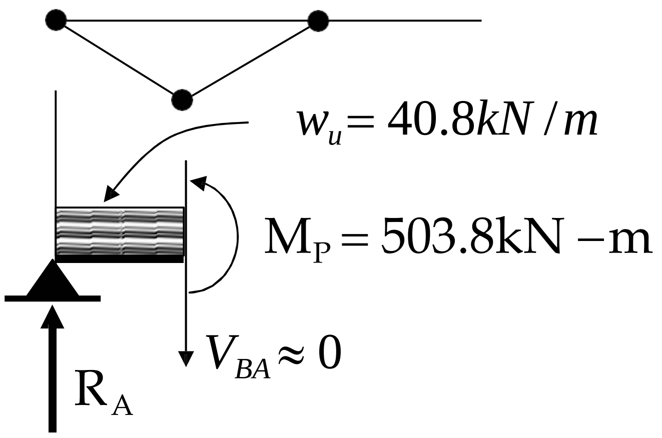
For interior span

$$-M_P + M_{s,available} = M_P$$

$$M_{s,available} = 2M_p > required$$

So exterior span is critical

$$R_A = \frac{40.8 \times 4.97}{2} + \frac{503.8}{4.97} = 202.8\text{kN}$$



Shear force just left of point C =
V_{CB} = Total load on span AB - R_A

$$= 40.8 \times 12 - 202.8 = 286.8\text{kN}$$

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No M_P is considered as hinge in not formed in interior span.

Solution: (contd…)

Shear force just right of point C = $V_{CD} = \dfrac{40.8 \times 12}{2} = 244.8\text{kN}$

$$Z_{req} = \frac{M_P}{\phi F_y} = \frac{503.8 \times 10^6}{0.9 \times 250}$$

$$= 2239 \times 10^3 mm^3$$

$$\phi_b M_n = \phi_b F_y \times 1.6 \times S_x \text{ ? Page 14}$$

$$= 0.9 \times 250 \times 1.6 \times 2150 \times 10^3 / 10^6$$

$$= 774 kN - m > 503.8 kN - m$$

$$d_{min} = \frac{L}{22} = \frac{12000}{22} \cong 545\text{mm}$$

Trial Section: W 610 x 92
(W530x92 has lesser depth and is not preferable)

$$\phi_v V_n = \phi_v \times \left(0.6 \times F_y\right) \times d \times t_w$$

$$= 0.9 \times \left(0.6 \times 250\right) \times 603 \times 10.9 / 1000$$

$$\phi_v V_n = 887.3 kN > V_{u\max} = 286.8 kN$$

Max. shear is present at first interior support

Beam is safe in shear for all sections

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Solution: (contd…)

<u>Stability Checks</u>

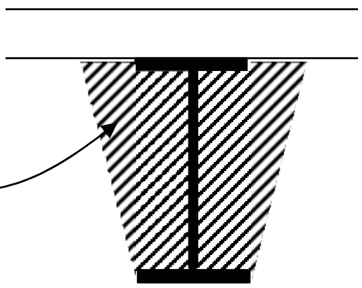
1. Web is continuously connected with flange

2. $\dfrac{b_f}{2t_f} = 6.0 < \lambda_P = 10.8$ **OK**

3. $\dfrac{h}{t_w} = 50.1 < \lambda_P = 107 \quad \text{as } P_u \approx 0$ **OK**

4. If continuously braced at top and vertical stiffeners are provided in –ve moment region then beam will be safe against LTB. (Lb<Lpd)

Stiffener for –ve moment region



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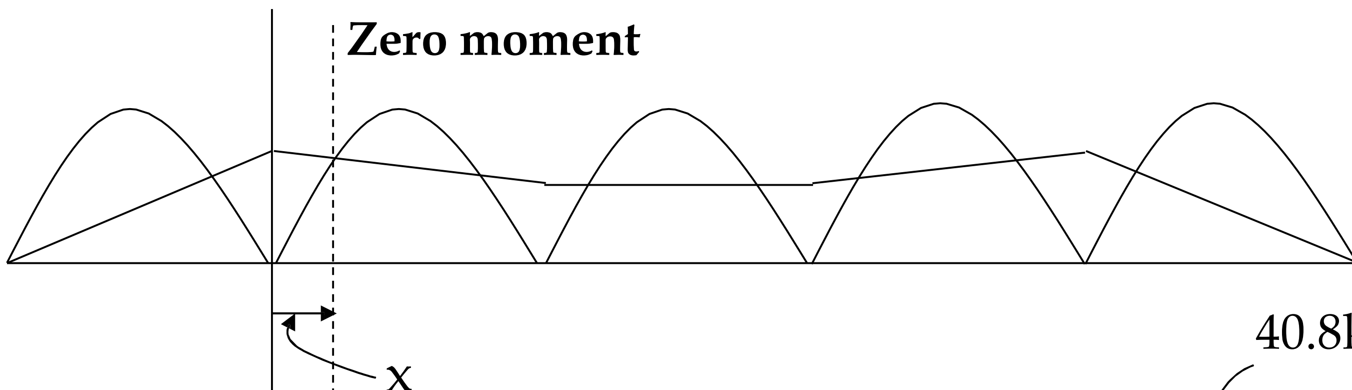
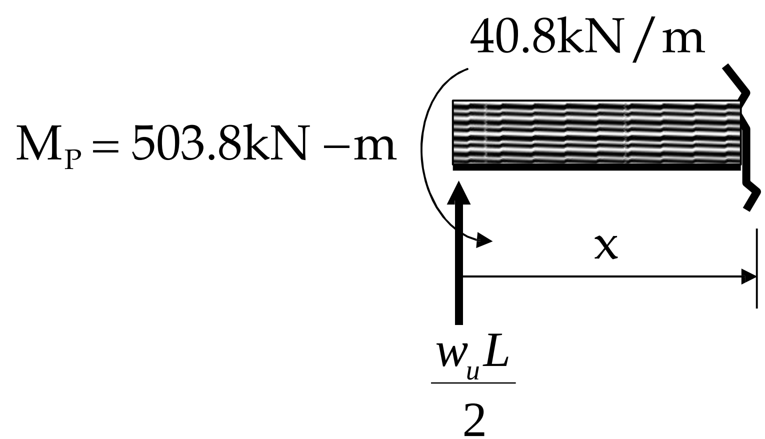
Solution: (contd…)

<u>Splices:</u>

Splicing is done in less critical region. e.g. Point of contraflexural





$$\frac{w_u L}{2}x - \frac{40.8x^2}{2} - 503.8 = 0$$

$$x = 9.3m, \quad 2.7m$$

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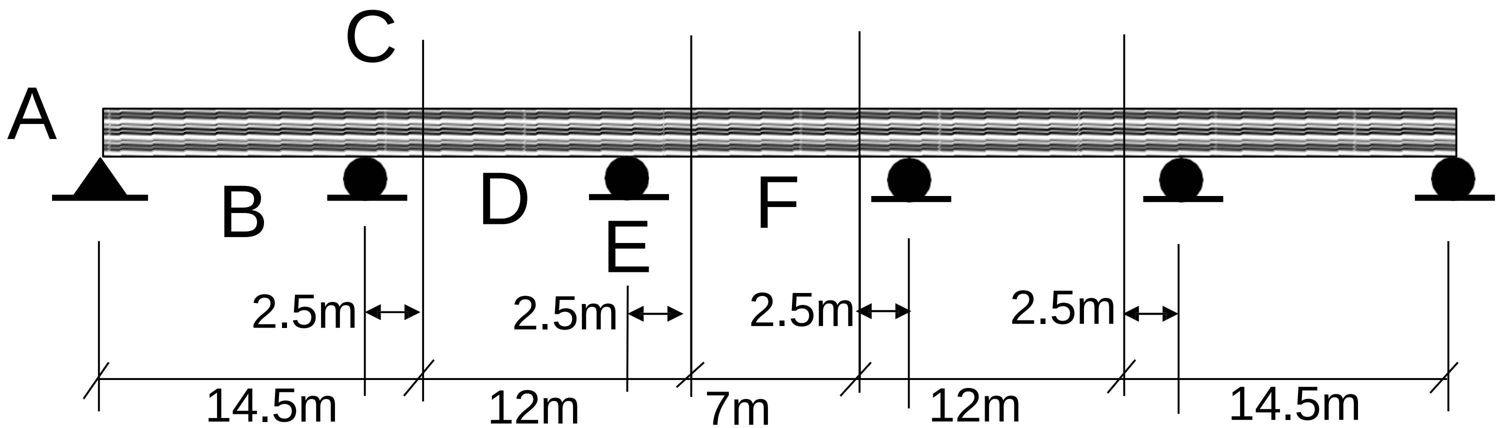
Splicing may be done as under:



Deflection at working load for end panel:

$$\Delta \approx \frac{w_L L^4}{185EI} = \frac{16.5 \times 12000^4}{185 \times 200000 \times 64500 \times 10^4} \cong 14.3mm$$

$$\Delta_{\max} \approx \frac{L}{360} = \frac{12000}{360} \cong 33mm \qquad \text{OK}$$

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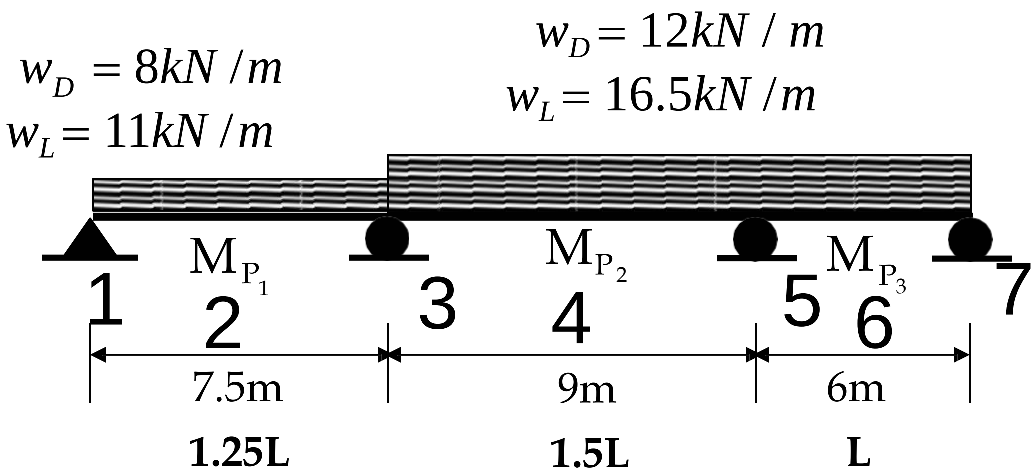
Example: Design continuous beam with different cross sections.



Solution:

$$w_{uL} = 1.2 \times 8 + 1.6 \times 11 = 27.2kN/m = w_u$$

$$w_{uR} = 1.2 \times 12 + 1.6 \times 16.5 = 40.8kN/m = 1.5w_u$$

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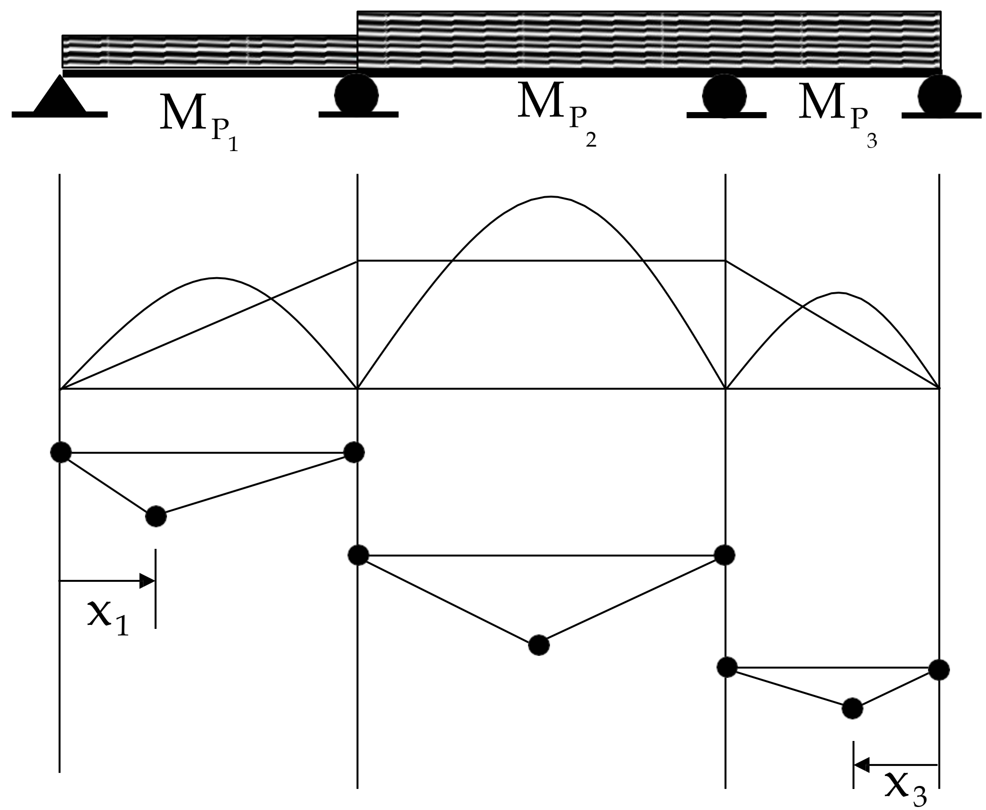
Suppose M_{P_2} will be largest. Extend this section into end spans on both sides and splice there. With this, at both interior supports M_{P_2} will exist.

$$M_{s1} = 191.25 kN - m$$

$$M_{s2} = 413.10 kN - m$$

$$M_{s3} = 183.60 kN - m$$



Central Panel

$$-M_{P_2} + M_{s_2} = M_{P_2}$$

$$M_{P_2} = \frac{M_{s_2}}{2}$$

$$\boxed{M_{P_2} = 206.55 kN - m}$$

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Left Panel

For end span we can't use the results of propped cantilever as on the support there is M_{P_2} while at mid-span its M_{P_1} or M_{P_3}

For positive hinge within the left span:

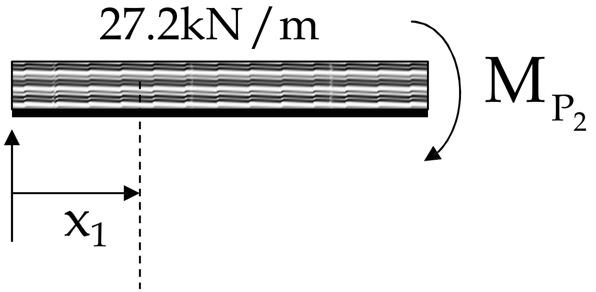
$$\frac{M_{p2}}{7.5}x_1 + M_{p1} = \frac{w_u \times 7.5}{2}x_1 - \frac{w_u}{2}x_1^{\,2}$$

$$M_{P_1} = \left(\frac{27.2\times 7.5}{2} - \frac{206.55}{7.5}\right)x_1 - \frac{27.2}{2}x_1^{\,2}$$

$$M_{P_1} = 74.46x_1 - 13.6x_1^{\,2}$$

$$\frac{dM_{P_1}}{dx_1} = 0 \Rightarrow x_1 = 2.7375\text{m}$$

$$\boxed{M_{P_1} = 101.9\text{kN}-\text{m}}$$



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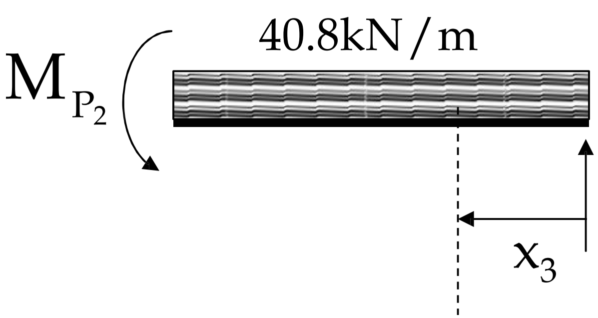
Right Panel

$$\frac{M_{p2}}{6}x_3 + M_{P_3} = \frac{1.5w_u \times 6}{2}x_3 - \frac{1.5w_u}{2}x_3^{\,2}$$

$$M_{P_3} = \left(\frac{40.8\times 6.0}{2} - \frac{206.55}{6.0}\right)x_3 - \frac{40.8}{2}x_3^{\,2}$$



$$M_{P_3} = -20.4x_3^{\,2} + 87.975x_3$$

$$\frac{dM_{P_3}}{dx_3} = 0 \Rightarrow x_3 = 2.156\text{m}$$

$$\boxed{M_{P_3} = 94.85\text{kN}-\text{m}}$$

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Solution: (contd…)

<u>Selection of Section</u>

Check that $\phi_b F_y x 1.6 S_x > M_{p,req}$

Span #	d_{min} (mm)	$Z_{req} = M_P/\phi F_y$ (mm^3)	Section Selected
1	341	453×10^3	W 360 x 32.9
2	409	918×10^3	W 460 x 52
3	272	422×10^3	W 310 x 32.7

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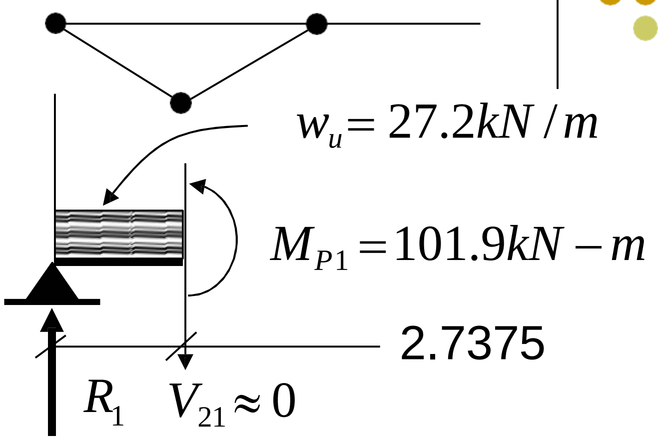
$$R_1 \times 2.7375 - \frac{27.2 \times 2.7375^2}{2} - 101.9 = 0$$

$$R_1 = 74.45 \text{ kN}$$



$$R_3 \approx V_{32} + V_{34}$$

$$= (w_u L_1 - R_1) + 1.5 w_u L_2 / 2$$

$$= 27.2 \times 7.5 - 74.45 + 40.8 \times 9 / 2$$

$$= 313.15 \text{ kN}$$

$$R_7 \times 2.156 - \frac{40.8 \times 2.156^2}{2} - 94.85 = 0$$

$$R_7 = 87.98 \text{ kN}$$

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$$R_5 \approx V_{56} + V_{54}$$

$$= (1.5w_u L_3 - R_7) + 1.5w_u L_2 / 2$$

$$= 40.8 \text{ x } 6 - 87.98 + 40.8 \text{ x } 9 / 2$$

$$= 340.42 \text{ kN}$$

The complete shear force diagram is thus shown in the next slide.

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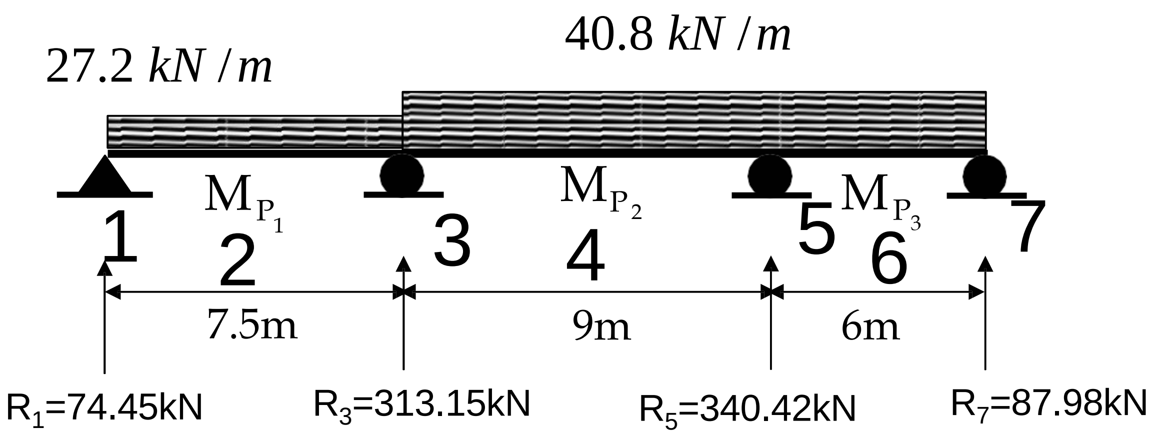

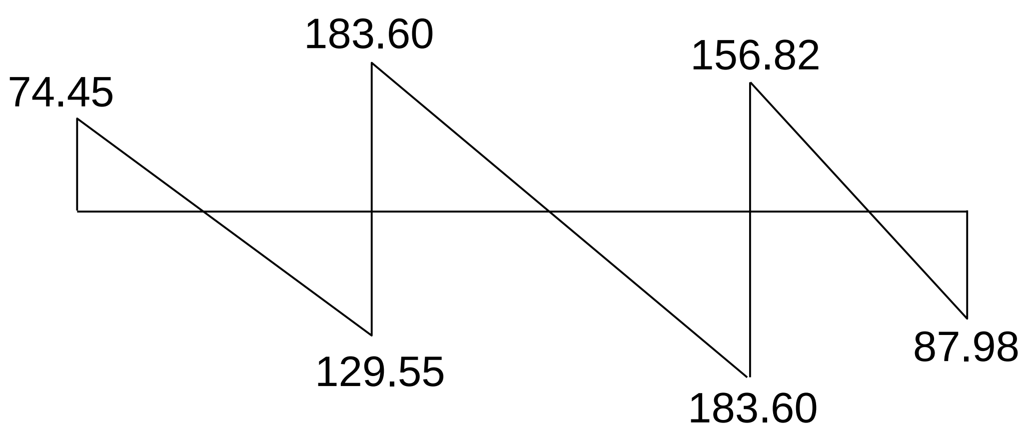

S. F. Diagram

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Solution: (contd…)

Shear Strength Check

Section	$\phi_v V_n$ (kN)	V_u (kN)	Remarks
W 360 x 32.9	273.3	129.5	**O.K.**
W 460 x 52	461.7	183.6	**O.K.**
W 310 x 32.7	278.9	156.82	**O.K.**

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Solution: (contd…)

<u>Check Section Stability</u>

Section	$b_f / 2t_f$ (10.8)	h / t_w (107)	Remarks
W 360 x 32.9	7.5	53.3	**O.K.**
W 460 x 52	7.1	41.8	**O.K.**
W 310 x 32.7	4.7	53.5	**O.K.**

Steel Structures

Solution: (contd…)

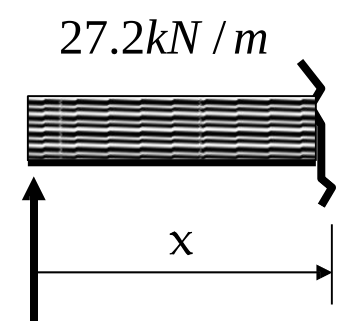
<u>Splices:</u>

Splicing is done in less critical region. e.g. Point of contraflexural

For left side:

$$M_x = \frac{w_u L_1}{2} x - \frac{w_u x^2}{2} - \frac{M_{p2}}{L_1} x$$

At point of contraflexure:



$$M_x = \frac{w_u L_1}{2} x - \frac{w_u x^2}{2} - \frac{M_{p2}}{L_1} x = 0$$

$$\frac{27.2 \times 7.5}{2} x - \frac{27.2 \times x^2}{2} - \frac{206.55}{7.5} x = 0$$

$$13.6 \times 7.5 - 13.6 \times x - \frac{206.55}{7.5} = 0 \qquad x = 5.5 \text{ m}$$

Similarly for the right hand side:

$$M_x = \frac{\square_u 1.5 w_3 L}{2} x - \frac{\square_u 1.5 w\, x^2}{2} - \frac{M_{p\,2}}{L_3} x = 0$$

$$122.4 - 20.4 \times x - 34.425 = 0 \qquad x = 4.3 \text{ m}$$

Provide splicing at the indicated spacing

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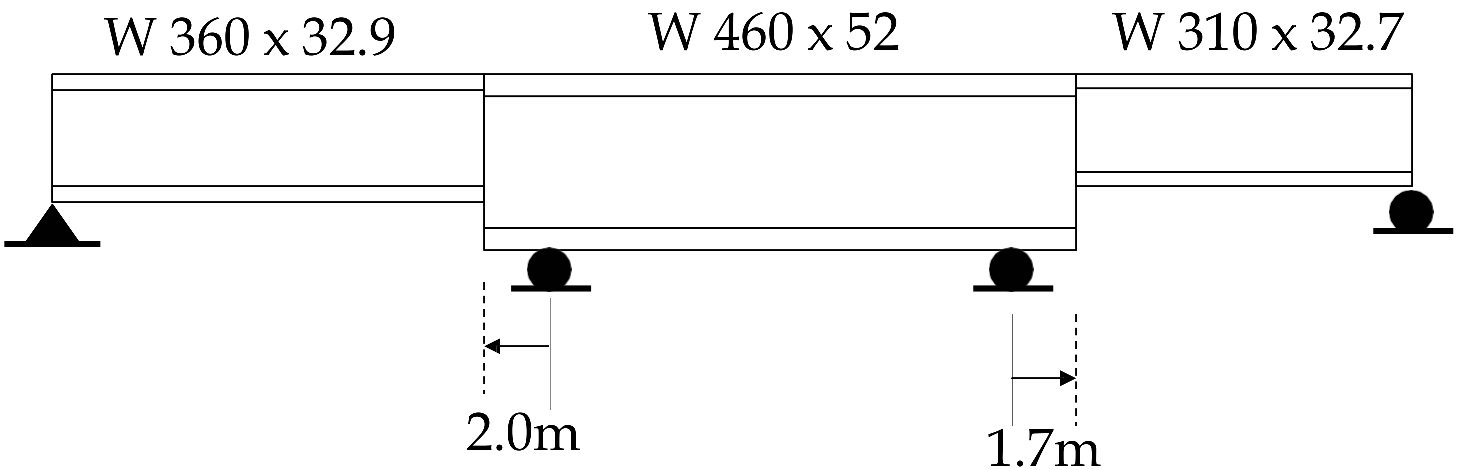
Solution: (contd…)

Splices



Bracing Requirements:

Assume continuous bracing at top and vertical plates in –ve moment region.

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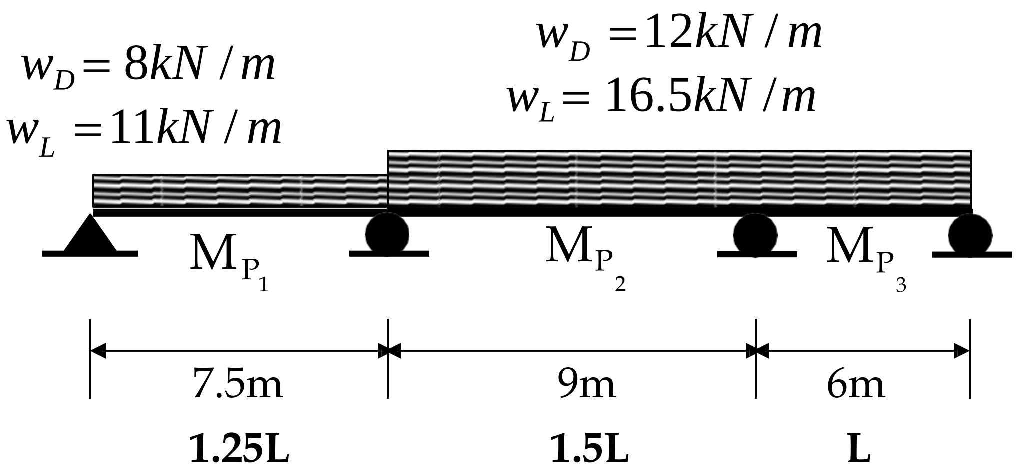
Example: Design continuous beam of the previous example using same cross section throughout using cover plates.



Solution:

$$w_{uL} = 1.2 \times 8 + 1.6 \times 11 = 27.2kN/m = w_u$$

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Steel Structures

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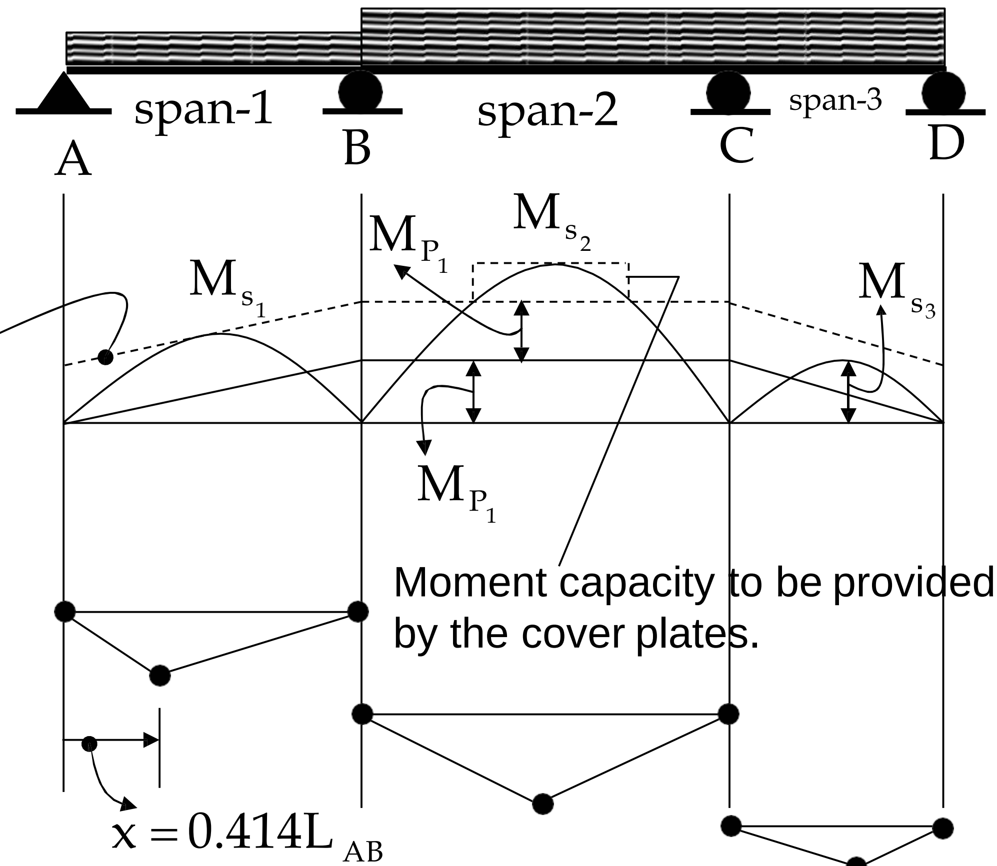

Capacity for any positive hinge to be developed. Central span has less capacity so BMD is going out of dashed line.

Select the section for span-1 and use the same throughout

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Solution: (contd…)

For span -1, like a propped cantilever beam.

$$M_{P_1} = 0.686 M_{s_1} = 131.2 kN - m$$

$$Z_{req} = \frac{M_{P_1}}{\phi F_Y} = \frac{131.2 \times 10^6}{0.9 \times 250}$$

$$= 583 \times 10^3\, mm^3$$

Check that $\phi_b F_y x 1.6 S_x > M_{p,req}$

$$d_{min} = \frac{900}{22} \cong 409 mm$$

Trial Section: W 410 x 38.8

$$Z = 724 \times 10^3\, mm^3 \qquad \phi_b M_P = 161 kN - m$$

$$\frac{b_f}{2t_f} = 8.0 < 10.8$$

$$\frac{h}{t_w} = 56.8 < 107$$

Steel Structures

Capacity of Right Panel

$$\frac{M_{p1}}{6}x_3 + M_{p3} = \frac{1.5w_u L_3}{2}x_3 - \frac{1.5w_u}{2}x_3^2$$

$$M_{p3} = -20.4x_3^2 + 122.40x_3 - 26.83x_3$$

For critical positive hinge location:

$$\frac{dM_{p3}}{dx_3} = -40.8x_3^2 + 95.57 = 0$$

x_3 = 2.46 m; M_{p3} = 111.9 kN-m < M_{p1} (OK)

Central Panel

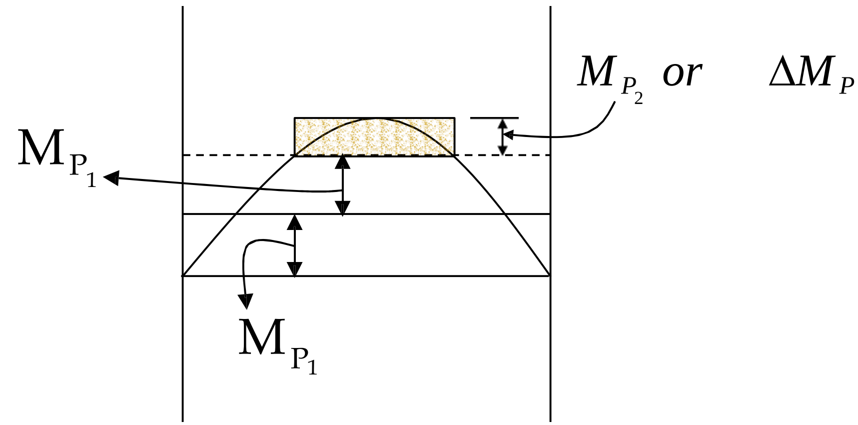
Let M_{p2} = design/safe moment capacity to be provided by the cover plates.

Steel Structures

For mid-span +ve hinge in span–2

$$M_{P_1} + M_{P_1} + M_{P_2} = M_{s_2}$$

$$M_{P_2} = M_{s_2} - 2M_{P_1}$$



$$M_{P_2} = 413.10 - 2 \times 161$$

$$M_{P_2} = 91.1\,\text{kN} - \text{m}$$

This is the value to be provided by cover plates

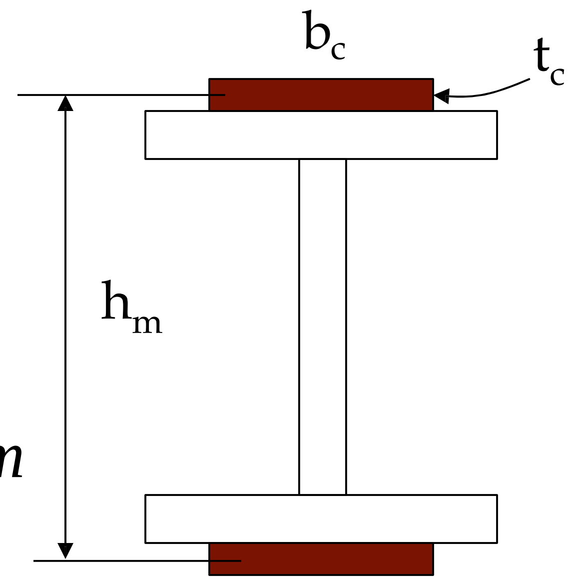
Steel Structures

Solution: (contd…)

Z provided by cover plates

$$b_c \times t_c \times h_m = \frac{M_{P_2}}{\phi F_y}$$

$$A_c = b_c \times t_c = \frac{M_{P_2}}{\phi F_y h_m}$$



$h_m = 399 + 6 = 405\text{mm}$

Let

$t_c = 6\text{mm}$ $\quad h_m = 399 + 6 = 405mm$

$$b_c = \frac{91.1 \times 10^6}{0.9 \times 250 \times 405 \times 6} = 167\text{mm}$$

b_f for trial section is 140 mm, so revise the size of cover plate

Let

Use

$t_c = 9\text{mm} \Rightarrow h_m = 399 + 9 = 408\text{mm} \Rightarrow \quad b_c = 111\text{mm} \cong 120\text{mm}$

Steel Structures

Solution: (contd…)

Use 9 x 120 mm flange cover plate on top and bottom.

$$\frac{b_c}{t_c} = \frac{120}{9} = 13.3 < 26.7 \quad \textbf{OK}$$

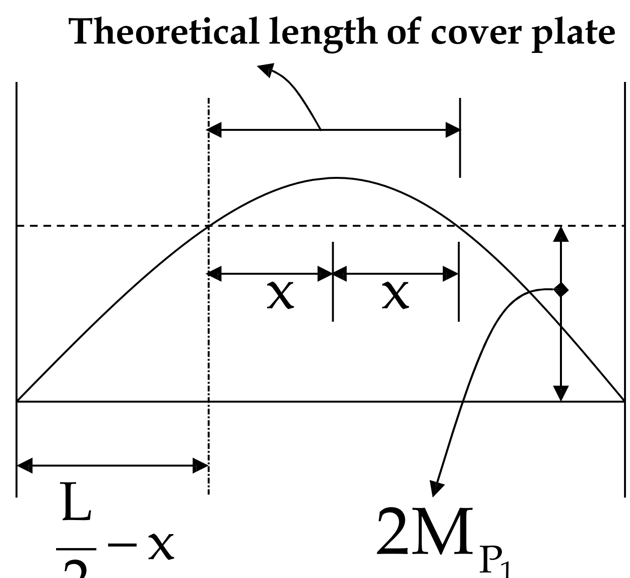
λ_P for stiffened flange, for A36 steel is 26.7

Length of Cover Plate:

M_s at point where M_{P_1} is just sufficient

$$= \frac{wL}{2}\left(\frac{L}{2} - x\right) - \frac{w(L/2 - x)^2}{2}$$

$$= \frac{w}{2}\left(\frac{L}{2} - x\right)\left[L - (L/2 - x)\right]$$



Steel Structures

Solution: (contd…)

$$= \frac{w}{2}\left(\frac{L}{2} - x\right)\left[L/2 + x\right]$$

$$= \frac{w}{2}\left(\frac{L^2}{4} - x^2\right)$$

$$= \frac{wL^2}{8}\left(1 - \frac{4x^2}{L^2}\right)$$

$$= M_{s_2}\left(1 - \frac{4x^2}{L^2}\right)$$

M_s at x is equal to $2M_{P1}$

$$M_{s_2}\left(1 - \frac{4x^2}{L^2}\right) = 2M_{P_1}$$

$$\frac{4x^2}{L^2} = 1 - \frac{2M_{P_1}}{M_{s_2}}$$

$$x = \frac{L}{2}\sqrt{1 - \frac{2M_{P_1}}{M_{s_2}}}$$

Length of cover plate is "2x"

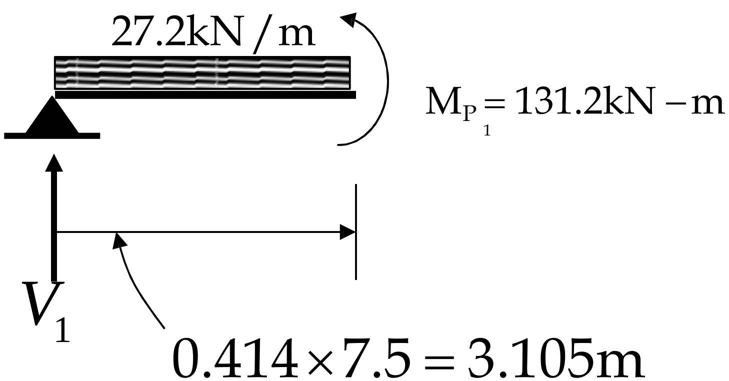
$$2x = 9.0\sqrt{1 - \frac{2 \times 161}{413.10}} \cong 4.25\text{m}$$

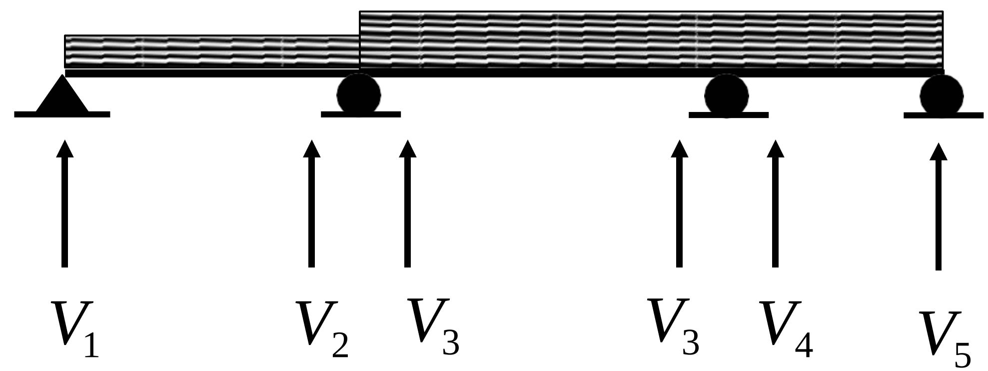
Provide 4.5m including 125 mm extra on both sides for development

Steel Structures

Solution: (contd…)

Shear Check:





We have provided 161kN-m instead of 131.2 kN-m and actually no full plastic hinge will form. Only partial hinge will be there.

$$V_1 = 84.5kN$$

$$V_2 = 119.5kN$$

$$V_3 = 183.6kN$$

$$V_4 = \frac{wL}{2} + \frac{M_P}{L} = 149.2kN$$

$$V_5 = \frac{wL}{2} - \frac{M_P}{L} = 95.6kN$$

$$\boxed{M_P = 161\text{kN}-\text{m}}$$

Steel Structures

Solution: (contd…)

Shear Check:

$$\phi_v V_n = 0.9 \times 0.6 \times 250 \times 399 \times 6.4 / 1000$$

$$\phi_v V_n = 344.7kN > V_{u\,\max} = 183.6kN$$

O.K.

Splices:

Splice for shear at inflection points as discussed in previous example.

Concluded