

Geometric Design and Highway Safety

TE-502 A/TE-502

Lecture-13

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VERTICAL ALIGNMENT

The vertical alignment of a transportation facility consists of tangent grades (straight lines in the vertical plane) and vertical curves.

Vertical alignment is documented by the profile.

The profile is a graph that has elevation as its vertical axis and distance, measured in stations along the centerline or other horizontal reference line of the facility, as its horizontal axis.

VERTICAL ALIGNMENT

Tangent Grades

Tangent grades are designated according to their slopes or grades. Maximum grades vary, depending on the type of facility, and usually do not constitute an absolute standard. The effect of a steep grade is to slow down the heavier vehicles (which typically have the lowest power/weight ratios) and increase operating costs.

Furthermore, the extent to which any vehicle (with a given power/weight ratio) is slowed depends on both the steepness and length of the grade. The effect of the slowing of the heavier vehicles depends on the situation, and is often more a matter of traffic analysis than simple geometric design. As a result, the maximum grade for a given facility is a matter of judgment, with the tradeoffs usually being cost of construction versus speed.

In the case of railroads, on the other hand, the tradeoff is an economic one, involving travel time, construction cost, and minimum power/weight ratios for trains on various grades.

VERTICAL ALIGNMENT

Tangent Grades

Table gives the maximum grades recommended for various classes of roadway by AASHTO. It should be understood that considerably steeper grades can be negotiated by passenger cars. In some urban areas, grades of minor streets may be as steep as 25 percent.

Recommended standards for maximum grades, percent

Type of terrain	Freeways	Rural highways	Urban highways
Level	3–4	3–5	5–8
Rolling	4–5	5–6	6–9
Mountainous	5–6	5–8	8–11

VERTICAL ALIGNMENT

Tangent Grades

3.4.1 Terrain

The topography of the land traversed has an influence on the alignment of roads and streets. Topography affects horizontal alignment, but has an even more pronounced effect on vertical alignment. To characterize variations in topography, engineers generally separate it into three classifications according to terrain—level, rolling, and mountainous.

In level terrain, highway sight distances, as governed by both horizontal and vertical restrictions, are generally long or can be made to be so without construction difficulty or major expense.

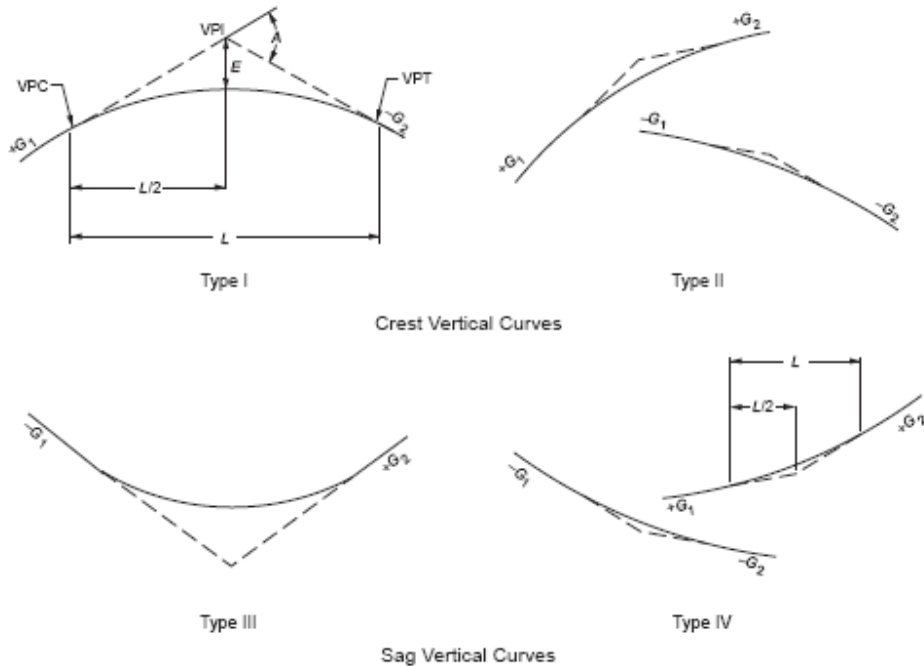
In rolling terrain, natural slopes consistently rise above and fall below the road or street grade, and occasional steep slopes offer some restriction to normal horizontal and vertical roadway alignment.

In mountainous terrain, longitudinal and transverse changes in the elevation of the ground with respect to the road or street are abrupt, and benching and side hill excavation are frequently needed to obtain acceptable horizontal and vertical alignment.

Terrain classifications pertain to the general character of a specific route corridor. Routes in valleys, passes, or mountainous areas that have all the characteristics of roads or streets traversing level or rolling terrain should be classified as level or rolling. In general, rolling terrain generates steeper grades than level terrain, causing trucks to reduce speeds below those of passenger cars; mountainous terrain has even greater effects, causing some trucks to operate at crawl speeds.

VERTICAL ALIGNMENT

Vertical Curves



G_1 and G_2 = Tangent Grades In Percent
 A = Algebraic Difference in Grade
 L = Length of Vertical Curve
 E = Vertical Offset at the VPI

Figure 3-41. Types of Vertical Curves

VERTICAL ALIGNMENT

Vertical Curves

Vertical tangents with different grades are joined by vertical curves such as the one shown in Figure. Vertical curves are normally parabolas centered about the point of intersection (P.I.) of the vertical tangents they join. Vertical curves are thus of the form

$$y = y_0 + g_1x + \frac{rx^2}{2}$$

where y = elevation of a point on the curve

y_0 = elevation of the beginning of the vertical curve (BVC)

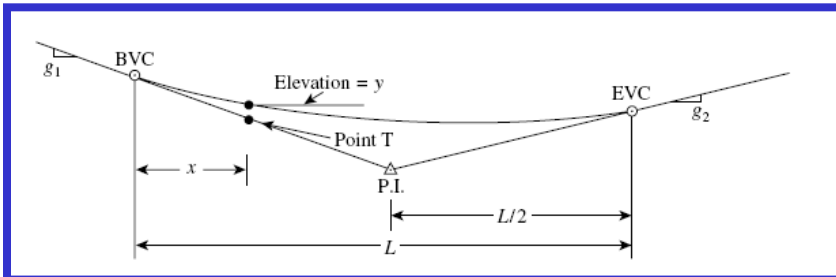
g_1 = grade just prior to the curve

x = horizontal distance from the BVC to the point on the curve

r = rate of change of grade

The rate of change of grade, in turn, is given by

$$r = \frac{g_2 - g_1}{L}$$



VERTICAL ALIGNMENT

Vertical Curves

where g_2 is the grade just beyond the end of the vertical curve (EVC) and L is the length of the curve. Also, vertical curves are sometimes described by K , the reciprocal of r . K is the distance in meters required to achieve a 1 percent change in grade. Vertical curves are classified as sags where $g_2 > g_1$ and crests otherwise. Note that r (and hence the term $rx^2/2$) will be positive for sags and negative for crests.

$$y = y_0 + g_1x + \frac{rx^2}{2}$$

where y = elevation of a point on the curve

y_0 = elevation of the beginning of the vertical curve (BVC)

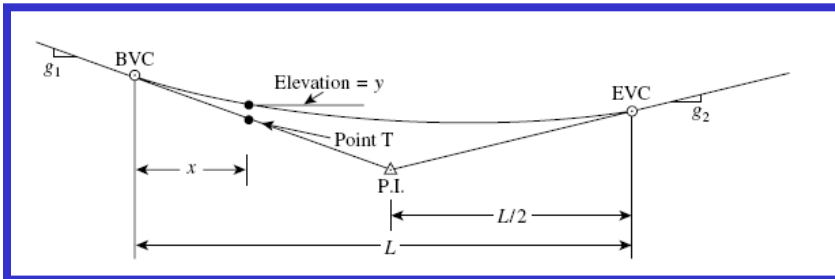
g_1 = grade just prior to the curve

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r = rate of change of grade

The rate of change of grade, in turn, is given by

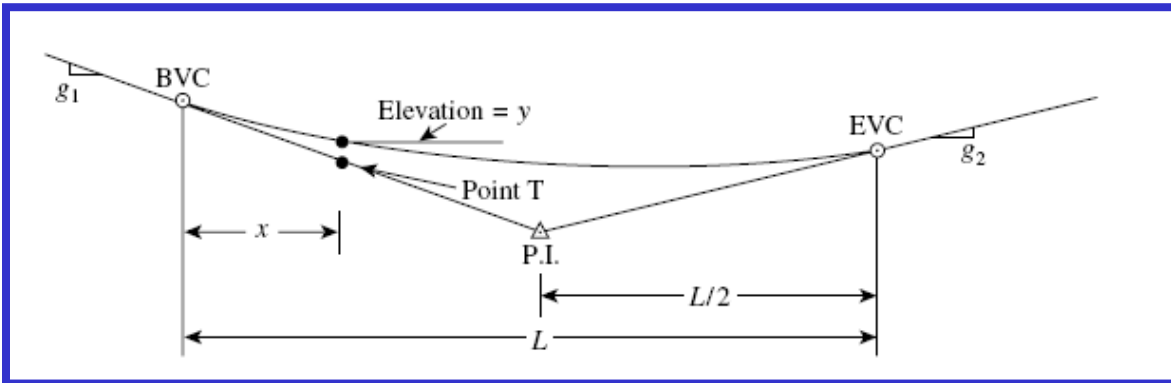
$$r = \frac{g_2 - g_1}{L}$$



VERTICAL ALIGNMENT

Vertical Curves

Also note that vertical distances in the vertical curve formulas are the product of grade times a horizontal distance. In consistent units, if vertical distances are to be in meters, horizontal distances should also be in meters, and grades should be dimensionless ratios. In many cases, however, it is more convenient to represent grades in percent and horizontal distance in stations. This produces the correct result, because the grade is multiplied by 100 and the horizontal distance divided by 100, and the two factors of 100 cancel. It is very important not to mix the two methods, however. If grades are in percent, horizontal distances *must be in stations*; likewise, if grades are dimensionless ratios, horizontal distances *must be in meters*.

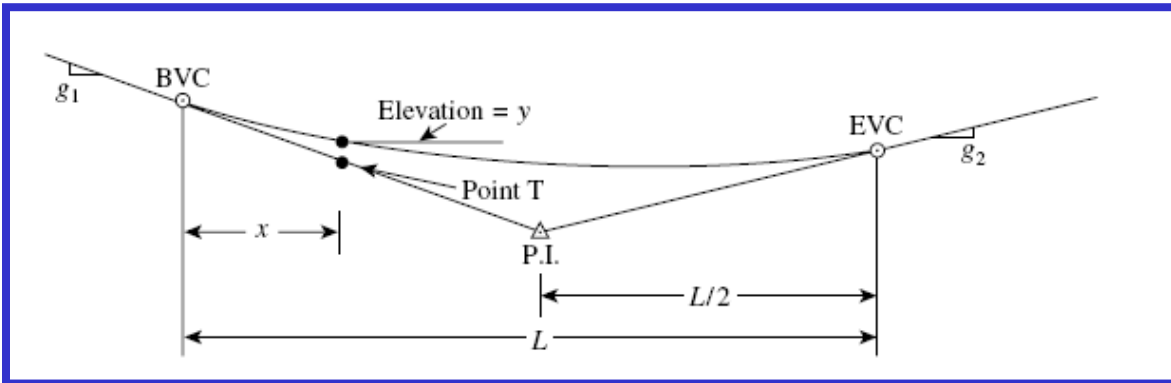


VERTICAL ALIGNMENT

Vertical Curves

The parabola is selected as the vertical curve so that the rate of change of grade, which is the second derivative of the curve, will be constant with distance. Note that the first derivative is the grade itself, and since the rate of change of grade is constant, the grade of any point in the vertical curve is a linear function of the distance from the BVC to the point. That is,

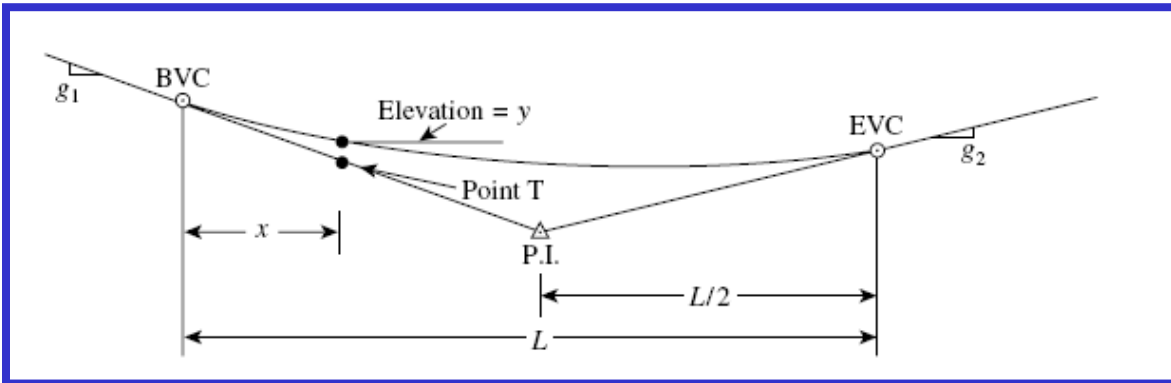
$$g = \frac{dy}{dx} = g_1 + rx$$



VERTICAL ALIGNMENT

Vertical Curves

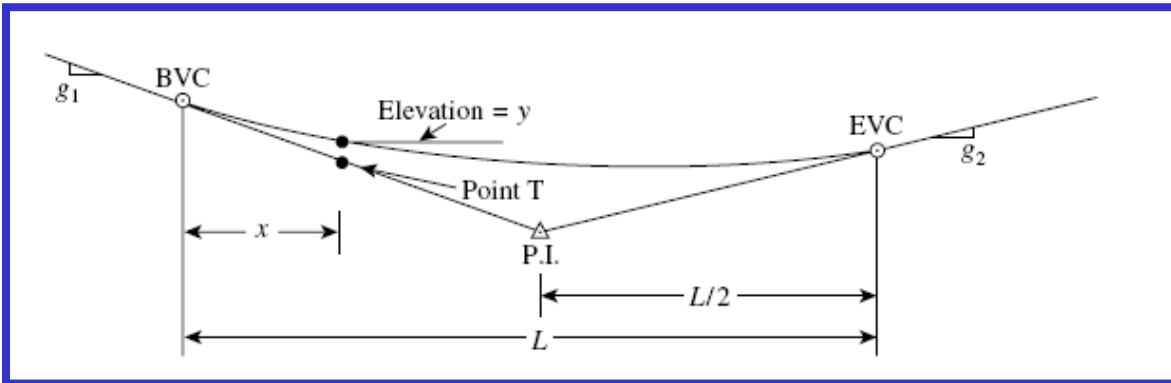
The quantity $rx^2/2$ is the distance from the tangent to the curve and is known as the offset. If x is always measured from the BVC, the offset given by $rx^2/2$ will be measured from the g_1 tangent. To determine offsets from the g_2 tangent, x should be measured backward from the EVC. Since the curve is symmetrical about its center, the offsets from the g_1 and g_2 tangents, respectively, are also symmetrical about the center of the curve, which occurs at the station of its P.I.



VERTICAL ALIGNMENT

Vertical Curves

Other properties of the vertical curve may be used to sketch it. For instance, at its center, the curve passes halfway between the P.I. and a chord joining the BVC and EVC. Normal drafting practice is to show the P.I. by means of a triangular symbol, as in Figure, although the extended vertical tangents shown in the figure are often omitted. The BVC and EVC are shown by means of circular symbols. The P.I., BVC, and EVC are identified by notes. The stations of the BVC and EVC are given in notes, as are the station and elevation of the P.I., the two tangent grades, and the length of the vertical curve.



VERTICAL ALIGNMENT

Vertical Curves-Sag curve-Numerical Problem

For a 300 m sag vertical curve between a 1.0% grade and a 6.0% grade [100+00 (150m) to 103+00], determine tangent elevations, offsets and profile elevations at stations placed at 25 m interval. **Please plot profile.**

Tangent elevation

Upto PI

BVC elevation+ g_1x

Beyond PI

PI elevation+ $g_2(x-L/2)$

Offset

$rx^2/2$

x measured from either

BVC or EVC

Profile elevation

Tangent elevation+offset

Tabular form for profile calculations in vertical curve

Station	Grade	Tangent elevation	Offset	Profile elevation
99 + 75	+1%	149.75		149.75
100 + 00	BVC	150.00		150.00
100 + 25		150.25	+0.05	150.30
100 + 50		150.50	+0.21	150.71
100 + 75		150.75	+0.47	151.22
101 + 00		151.00	+0.83	151.83
101 + 25		151.25	+1.30	152.55
101 + 50	PI.	151.50	+1.88	153.38
101 + 75		153.00	+1.30	154.30
102 + 00		154.50	+0.83	155.33
102 + 25		156.00	+0.47	156.47
102 + 50		157.50	+0.21	157.71
102 + 75		159.00	+0.05	159.00
103 + 00	EVC	160.50		160.50
103 + 25	+6%	162.00		162.00

VERTICAL ALIGNMENT

Vertical Curves-Numerical Problem

A -2.5% grade is connected to a +1.0% grade by means of a 180 m vertical curve. The P.I. station is 100 + 00 and the P.I. elevation is 100.0 m above sea level. What are the station and elevation of the lowest point on the vertical curve?

Rate of change of grade:

$$r = \frac{g_2 - g_1}{L} = \frac{1.0\% - (-2.5\%)}{1.8 \text{ sta}} = 1.944\%/sta$$

Station of the low point:

At low point, $g = 0$

$$g = g_1 + rx = 0$$

or

$$x = \frac{-g_1}{r} = -\left(\frac{-2.5}{1.944}\right) = 1.29 = 1 + 29 \text{ sta}$$

$$\text{Station of BVC} = (100 + 00) - (0 + 90) = 99 + 10$$

$$\text{Station of low point} = (99 + 10) + (1 + 29) = 100 + 39$$

VERTICAL ALIGNMENT

Vertical Curves-Numerical Problem

A -2.5% grade is connected to a +1.0% grade by means of a 180 m vertical curve. The P.I. station is 100 + 00 and the P.I. elevation is 100.0 m above sea level. What are the station and elevation of the lowest point on the vertical curve?

Elevation of BVC:

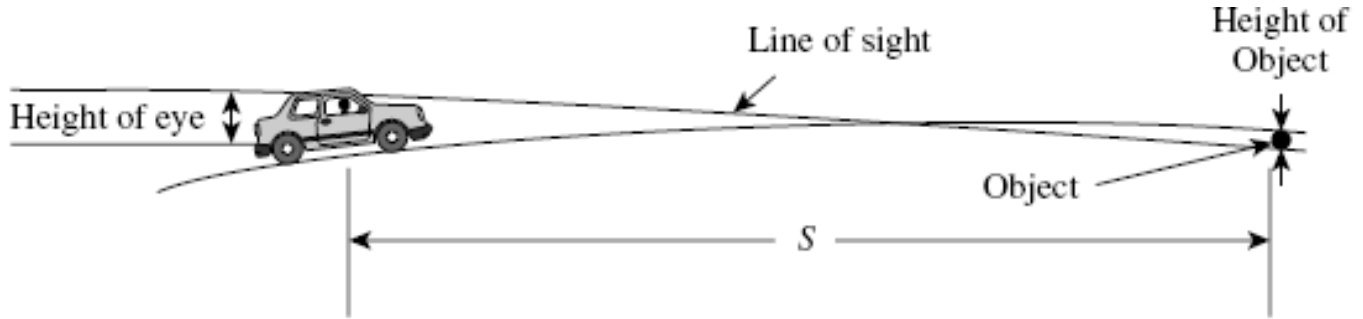
$$y_0 = 100.0 \text{ m} + (-0.9 \text{ sta})(-2.5\%) = 102.25 \text{ m}$$

Elevation of low point:

$$\begin{aligned} y &= y_0 + g_1x + \frac{rx^2}{2} \\ &= 102.25 \text{ m} + (-2.5\%)(1.29 \text{ sta}) + \frac{(1.944\%/sta)(1.29 \text{ sta})^2}{2} \\ &= 100.64 \text{ m} \end{aligned}$$

VERTICAL ALIGNMENT

Vertical Curves-SSD for Crests



Stopping sight distance diagram for crest vertical curve.

VERTICAL ALIGNMENT

Vertical Curves-SSD for Crests

$$L_{\min} = \begin{cases} \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} & \text{When } S < L \\ 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A} & \text{When } S > L \end{cases}$$

where S = sight distance

L = vertical curve length

A = absolute value of the algebraic difference in grades, in percent, $|g_1 - g_2|$

h_1 = height of eye

h_2 = height of object

For stopping sight distance, the height of object is normally taken to be 0.150 m. For passing sight distance, the height of object used by AASHTO is 1.300 m. Height of eye is assumed to be 1.070 m.

VERTICAL ALIGNMENT

Vertical Curves-SSD for Sags

$$L_{\min} = \begin{cases} \frac{AS^2}{200[0.6 + S(\tan 1^\circ)]} = \frac{AS^2}{120 + 3.5S} & \text{When } S < L \\ 2S - \frac{200[0.6 + S(\tan 1^\circ)]}{A} = 2S - \frac{120 + 3.5S}{A} & \text{When } S > L \end{cases}$$

VERTICAL ALIGNMENT

Vertical Curves

Minimum vertical curve standards for highways may also be based on appearance. This problem arises because short vertical curves tend to look like kinks when viewed from a distance. Appearance standards vary from agency to agency. Current California standards, for instance, require a minimum vertical curve length of 60 m where grade breaks are less than 2 percent or design speeds are less than 60 km/h. Where the grade break is greater than 2 percent and the design speed is greater than 60 km/h, the minimum vertical curve is given by $L = 2V$, where L is the vertical curve length in meters and V is the design speed in km/h.

VERTICAL ALIGNMENT

Vertical Curves-Numerical Problem

Determine the minimum length of a crest vertical curve between a +0.5% grade and a -1.0% grade for a road with a 100-km/h design speed. The vertical curve must provide 190-m stopping sight distance and meet the California appearance criteria. Round up to the next greatest 20 m interval.

Stopping sight distance criterion:

Assume $S < L$

$$L = \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{[0.5 - (-1.0)](190^2)}{200(\sqrt{1.070} + \sqrt{0.150})^2} = 134.0 \text{ m}$$

134.0 m < 190 m, so $S > L$

$$\begin{aligned} L &= 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A} = 2(190) - \frac{200(\sqrt{1.070} + \sqrt{0.150})^2}{[0.5 - (-1.0)]} \\ &= 380.0 - 269.5 = 110.5 \text{ m} \end{aligned}$$

VERTICAL ALIGNMENT

Vertical Curves-Numerical Problem

Determine the minimum length of a crest vertical curve between a 0.5% grade and a 1.0% grade for a road with a 100-km/h design speed. The vertical curve must provide 190-m stopping sight distance and meet the California appearance criteria. Round up to the next greatest 20 m interval.

Appearance criterion:

Design speed = 100 km/h > 60 km/h but grade break = 1.5% < 2%. Use 60 m.

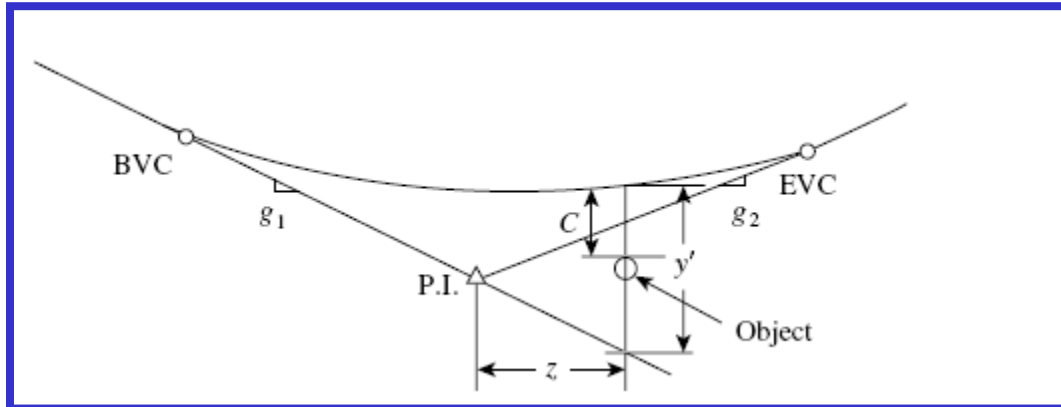
Conclusion:

Sight distance criterion governs. Use 120 m vertical curve.

VERTICAL ALIGNMENT

Vertical Curves-Clearance over or under Objects

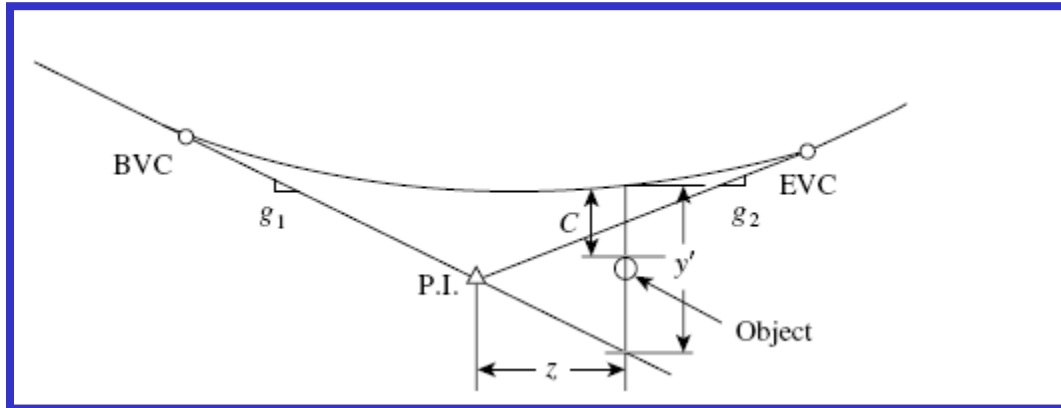
Vertical curve lengths may be limited by the need to provide clearances over or under objects such as overpasses or drainage structures. In the case of sag vertical curves passing over objects or crest vertical curves passing under them, the required clearances establish minimum lengths; in the case of crest vertical curves passing over objects or sags passing under them, the clearances establish maximum lengths. Where clearances limit vertical curve lengths, adequate sight distance should still be provided.



VERTICAL ALIGNMENT

Vertical Curves-Clearance over or under Objects

In either case, the maximum or minimum length of the vertical curve may be determined by assuming that the clearance is barely met and calculating the length of the vertical curve passing through the critical point thus established. It is easiest to do this as illustrated by Figure. In the figure, C represents the critical clearance, z the horizontal distance from the P.I. to the critical point, and y' the offset between the critical point and the tangent passing through the BVC.



VERTICAL ALIGNMENT

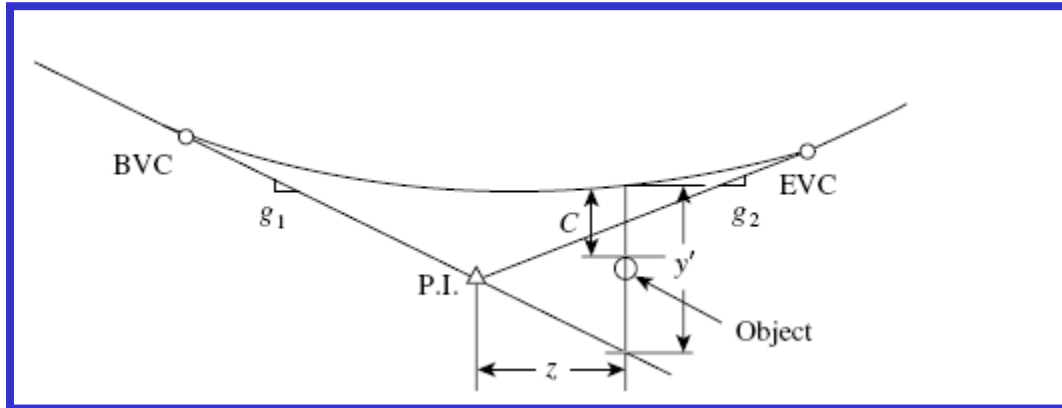
Vertical Curves-Clearance over or under Objects

The equation for the offset is

$$y' = \frac{rx^2}{2}$$

$$r = \frac{g_2 - g_1}{L} = \frac{A}{L}$$

$$x = \frac{L}{2} + z$$

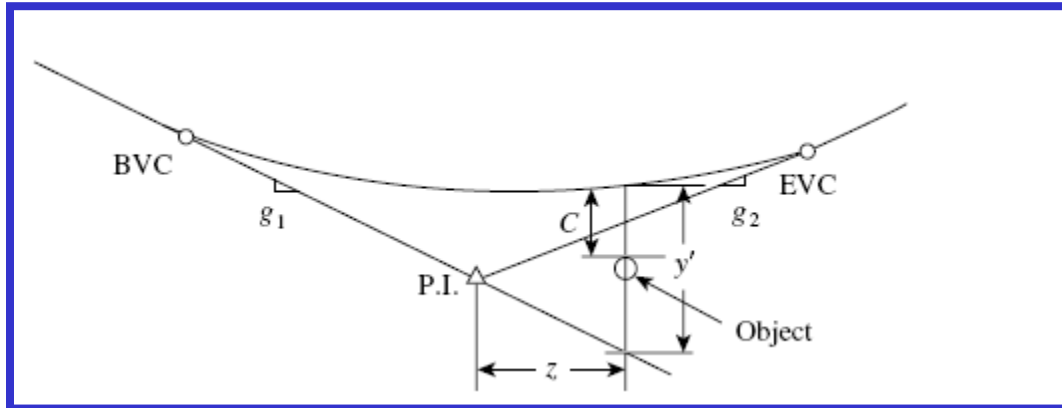


VERTICAL ALIGNMENT

Vertical Curves-Clearance over or under Objects

$$y' = \frac{A(L/2 + z)^2}{2L}$$

$$AL^2 + (4Az - 8y')L + 4Az^2 = 0$$



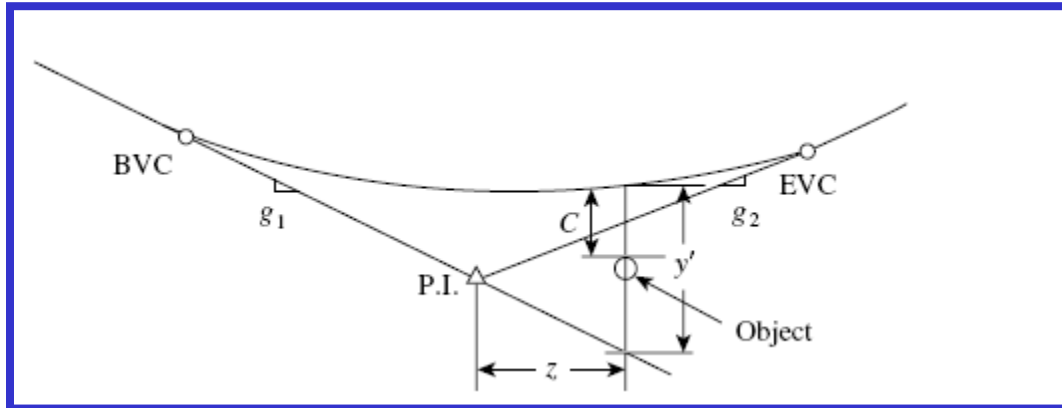
VERTICAL ALIGNMENT

Vertical Curves-Clearance over or under Objects

$$AL^2 + (4Az - 8y')L + 4Az^2 = 0$$

Taking $w = y'/A$

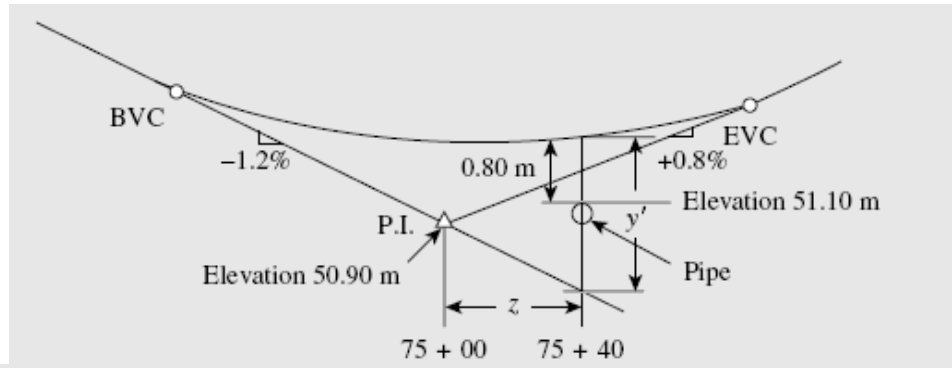
$$L = 4w - 2z + 4\sqrt{w^2 - wz}$$



VERTICAL ALIGNMENT

Vertical Curves-Clearance over or under Objects-Numerical Problem

A vertical curve joins a -1.2% grade to a $+0.8\%$ grade. The P.I. of the vertical curve is at station $75 + 00$ and elevation 50.90 m above sea level. The centerline of the roadway must clear a pipe located at station $75 + 40$ by 0.80 m. The elevation of the top of the pipe is 51.10 m above sea level. What is the minimum length of the vertical curve that can be used?



Determine z :

$$z = (75 + 40) - (75 + 00) = 0.40 \text{ sta.}$$

Determine y'

$$\text{Elevation of tangent} = 50.90 + (-1.2)(0.4) = 50.42 \text{ m}$$

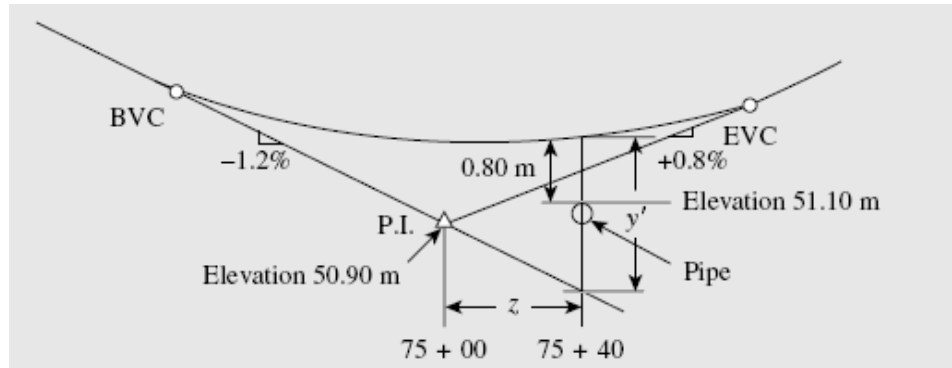
$$\text{Elevation of roadway} = 51.10 + 0.80 = 51.90 \text{ m}$$

$$y' = 51.90 - 50.42 = 1.48 \text{ m}$$

VERTICAL ALIGNMENT

Vertical Curves-Clearance over or under Objects-Numerical Problem

A vertical curve joins a -1.2% grade to a $+0.8\%$ grade. The P.I. of the vertical curve is at station $75 + 00$ and elevation 50.90 m above sea level. The centerline of the roadway must clear a pipe located at station $75 + 40$ by 0.80 m. The elevation of the top of the pipe is 51.10 m above sea level. What is the minimum length of the vertical curve that can be used?



Determine w :

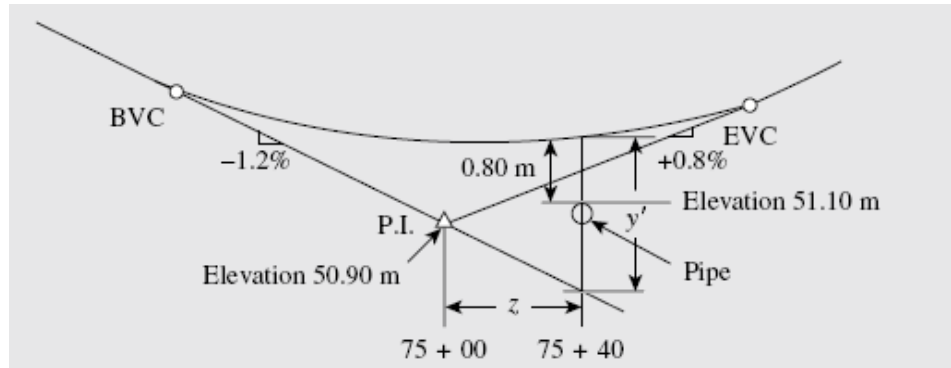
$$A = g_2 - g_1 = (+0.8) - (-1.2) = 2.0$$

$$w = \frac{y'}{A} = \frac{1.48}{2} = 0.74$$

VERTICAL ALIGNMENT

Vertical Curves-Clearance over or under Objects-Numerical Problem

A vertical curve joins a -1.2% grade to a $+0.8\%$ grade. The P.I. of the vertical curve is at station $75 + 00$ and elevation 50.90 m above sea level. The centerline of the roadway must clear a pipe located at station $75 + 40$ by 0.80 m. The elevation of the top of the pipe is 51.10 m above sea level. What is the minimum length of the vertical curve that can be used?



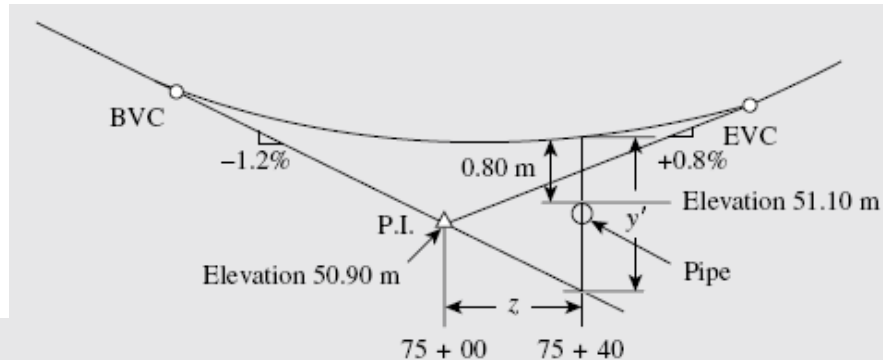
Determine L :

$$L = 4w - 2z + 4\sqrt{w^2 - wz}$$
$$= 4(0.74) - 2(0.4) + 4\sqrt{0.74^2 - (0.74)(0.4)} = 4.17 \text{ sta} = 417 \text{ m}$$

VERTICAL ALIGNMENT

Vertical Curves-Clearance over or under Objects-Numerical Problem

A vertical curve joins a -1.2% grade to a $+0.8\%$ grade. The P.I. of the vertical curve is at station $75 + 00$ and elevation 50.90 m above sea level. The centerline of the roadway must clear a pipe located at station $75 + 40$ by 0.80 m. The elevation of the top of the pipe is 51.10 m above sea level. What is the minimum length of the vertical curve that can be used?



Check y' :

$$x = \frac{4.17}{2} + 0.4 = 2.485 \text{ sta}$$

$$r = \frac{A}{L} = \frac{2}{4.17} = 0.48$$

$$y' = \frac{rx^2}{2} = \frac{(0.48)(2.485^2)}{2} = 1.48$$

COORDINATION OF VERTICAL AND HORIZONTAL ALIGNMENTS

Transportation facilities such as highways and railways are three-dimensional objects. Although many aspects of their design can be determined by considering horizontal and vertical alignment separately from one another, it is important to understand the relationship between them.

Proper coordination of horizontal and vertical alignment is important for reasons related to the esthetics, economics, and safety of the facility.

As a general rule, horizontal curvature and grades should be kept in balance. That is, the designer should avoid both the provision of minimal curvature at the expense of long, steep grades and the provision of level vertical alignment at the expense of excessive horizontal curvature.

COORDINATION OF VERTICAL AND HORIZONTAL ALIGNMENTS

Where there is both horizontal and vertical curvature, it is normally best from an esthetic standpoint to provide the impression of a single three-dimensional curve in both the horizontal and vertical planes. This means that horizontal and vertical curves should normally coincide. In some cases, however, safety considerations may suggest that horizontal curves be extended beyond vertical curves in order to avoid hiding the beginning of the horizontal curve from drivers approaching it. This is especially important where horizontal curves coincide with rather sharp crest vertical curves.

In addition, use of long, relatively gentle curves and short tangents will normally produce a more flowing line than will use of long tangents and short, sharp curves. This is especially true when the deflection angle of horizontal tangents or the difference between grades is small. This effect is illustrated by Figure.

COORDINATION OF VERTICAL AND HORIZONTAL ALIGNMENTS



a. Long curves, short tangents



b. Short curves, long tangents

FIGURE

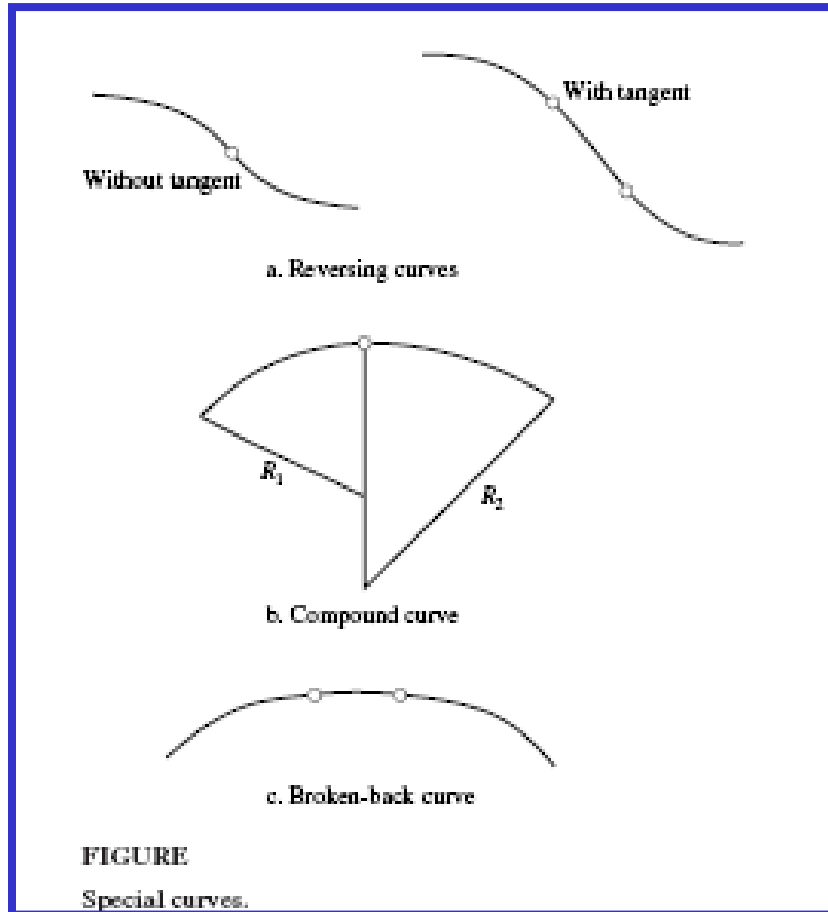
Long curves and short tangents provide a more fluid alignment.

COORDINATION OF VERTICAL AND HORIZONTAL ALIGNMENTS

At the same time, too much curvature may also pose problems. On two-lane highways, for instance, it is also important to provide an adequate number of sections with passing sight distance, and these sections need to be of adequate length to prevent drivers from becoming impatient when following slow-moving vehicles. Finally, on intersection approaches, highway alignments should provide adequate sight distance and should be as flat and straight as possible.

In addition to these rules, there are a number of guidelines related to the positioning of curves (either horizontal or vertical) relative to one another. These are based on esthetics and in the case of horizontal curves, on the necessity of providing adequate superelevation transitions. Figure illustrates several special curve combinations.

COORDINATION OF VERTICAL AND HORIZONTAL ALIGNMENTS



COORDINATION OF VERTICAL AND HORIZONTAL ALIGNMENTS

***Reversing horizontal curves* must be separated by a tangent or by transition curves to allow for the development of superelevation.**

The superelevated cross-slopes of the roadway will be in opposite directions in the two curves, so that some distance must be provided for rotation of the cross section. Where transition curves are used, however, the ST of the first curve may coincide with the TS of the second. *Reversing vertical curves* pose no problem.

***Compound curves* result when two curves of differing radius (or for vertical curves, different rates of change of grade) join one another. Such curves are normally avoided for centerline alignment, although use of three-centered compound curves for pavement edges in intersections is common. Problems with compound curves include esthetics and, in the case of horizontal curves, difficulty in developing the necessary superelevation transition, as well as possible deception of the driver as to the severity of the curve.**

COORDINATION OF VERTICAL AND HORIZONTAL ALIGNMENTS

Compound curves may be acceptable if the difference in radius is small or if they occur on a one-way roadway and the radius of curvature increases in the direction of travel.

***Broken-back curves* consist of two curves in the same direction separated by a short tangent. Such curves (whether horizontal or vertical) are objectionable on esthetic grounds and should be replaced by a single, larger-radius curve.**